hard (i.e., the Nash-extension problem in general IDS games is NP-complete [Kearns and Ortiz, 2003]). We do not know of any other non-trivial game for which there exists a polynomial-time algorithm to compute all NE except ours and the algorithm for uniform-transfer IDS games of Kearns and Ortiz [2003].

In Section 4, we provide experimental results from applying learning-in-games heuristics to compute approximate NE to both fixed and randomly-generated instances of IDD games, with at most one simultaneous attack and one-hop transfers, on a very large Internet AS graph ($\approx 27K$ nodes and $\approx 100K$ edges).

### 2. IDS GAMES

Each player $i$ in an IDS game has a choice to invest ($a_i = 1$) or not invest ($a_i = 0$). For each player $i$, $C_i$ and $L_i$ are the cost of investment and loss induce by the bad event, respectively. We define the ratio of the two parameters, the player’s “cost-to-loss” ratio, as $p_i \equiv C_i/L_i$. Bad events can occur through both direct and indirect means. Direct risk, or internal risk, $p_i$ is the probability that player $i$ will experience a bad event because of direct contamination. The standard IDS model assumes that investing will completely protect the player from direct contamination; hence, internal risk is only possible when $a_i = 0$. Indirect risk $q_{ij}$ is the probability that player $j$ is directly “contaminated,” does not experience the bad event but transfers it to player $i$ who ends up experiencing the bad event. There is an implicit global constraint on these parameters, by the axioms of probability: $p_i + \sum_{j=1}^n q_{ij} \leq 1$ for all $i$.

We now formally define a (directed) graphical games [Kearns, 2007, Kearns et al., 2001] version of IDS games, as first introduced by Kearns and Ortiz [2003]. Denote by $[n] \equiv \{1, \ldots, n\}$ the set of $n$ players. Note that the parameters $q_{ij}$’s induce a directed graph $G = ([n], E)$ such that $E \equiv \{(i,j)\mid q_{ij} > 0\}$. Let $Pa(i) \equiv \{j \mid q_{ji} > 0\}$ be the set of players that are parents of player $i$ in $G$ (i.e., the set of players that player $i$ is exposed to via transfers), and by $PF(i) \equiv Pa(i) \cup \{i\}$ the parent family of player $i$, which includes $i$. Denote by $k_i \equiv |PF(i)|$ the size of the parent family of player $i$. Similarly, let $Ch(i) \equiv \{j \mid q_{ij} > 0\}$ be the set of players that are children of player $i$ (i.e., the set of players to whom player $i$ can present a risk via transfer) and $CF(i) \equiv Ch(i) \cup \{i\}$ the (children) family of player $i$, which includes $i$. The probability that player $i$ is safe from player $j$, as a function of player $j$’s decision, is

$$e_{ij}(a_j) \equiv a_j + (1-a_j)(1-q_{ji}) = (1-q_{ji})^{1-a_j},$$

because if $j$ invests, then it is impossible for $j$ to transfer the bad event, while if $j$ does not invest, then $j$ either experiences the bad event or transfers it to another player, but never both.

Denote by $a \equiv (a_1, \ldots, a_n) \in \{0,1\}^n$ the joint action of all $n$ players. Also denote by $a_{-i}$ the joint-action of all players except $i$ and for any subset $I \subset [n]$ of players, denote by $a_I$ the sub-component of the joint action corresponding to those players in $I$ only. We define $i$’s overall safety from all other players as $s_i(a_{Pa(i)}) \equiv \prod_{j \in Pa(i)} e_{ij}(a_j)$ and equivalently his overall risk from some other players is $r_i(a_{Pa(i)}) \equiv 1 - s_i(a_{Pa(i)})$. Note that each players’ external safety (and risk) is a direct function of its parents only, not all other players. From these definitions, we obtain player $i$’s overall cost, the cost of joint action $a \in \{0,1\}^n$, corresponding to the (binary) investment decision of all players, is

$$M_i(a_i, a_{Pa(i)}) \equiv a_i[C_i + r_i(a_{Pa(i)})L_i] + (1-a_i)[p_i + (1-p_i)r_i(a_{Pa(i)})]L_i .$$

### 3. IDD GAMES

In the standard IDS game model, investment in security does not reduce transfer risks. However, in some IDS settings (e.g., vaccination and cyber-security), it is reasonable to expect that security investments would include mechanisms to reduce transfer risks. This motivates our first modification to the traditional IDS games: allowing the investment in protection to not reduce (or even eliminate) the transfer risk. We incorporate this factor by introducing a new real-valued parameter $\alpha_i \in [0,1]$ representing the probability that a transfer of a potentially bad event will go unblocked by $i$’s security, assuming $i$ has invested. Thus, we redefine player $i$’s overall cost as

$$M_i(a_i, a_{Pa(i)}) \equiv a_i[C_i + \alpha_i r_i(a_{Pa(i)})L_i] + (1-a_i)[p_i + (1-p_i)r_i(a_{Pa(i)})]L_i .$$

Next, we introduce an additional player, the attacker, who deliberately initiates bad events. (So that now bad events are no longer “chance occurrences” without any strategic deliberation.) The attacker has a target decision for each player - a choice of attack ($b_i = 1$) or not attack ($b_i = 0$) player $i$. Hence, a similar extension was also proposed independently by Heal and Kunreuther [2007].