so we’re right back where we started. This is probably not what the framers of
the identity intended; but it’s reassuring to know that we haven’t gone astray.

Some applications of (5.14) are, of course, more useful than this. We can
use upper negation, for example, to move quantities between upper and lower
index positions. The identity has a symmetric formulation,
\[
(-1)^m\binom{-n-1}{m} = (-1)^n\binom{-m-1}{n}, \quad \text{integers } m, n \geq 0, \quad (5.15)
\]
which holds because both sides are equal to \(\binom{m+n}{n}\).

Upper negation can also be used to derive the following interesting sum:
\[
\sum_{k \leq m} \binom{r}{k}(-1)^k = \binom{r}{0} - \binom{r}{1} + \cdots + (-1)^m\binom{r}{m}
\]
\[
= (-1)^m\binom{r-1}{m}, \quad \text{integer } m. \quad (5.16)
\]
The idea is to negate the upper index, then apply (5.9), and negate again:
\[
\sum_{k \leq m} \binom{r}{k}(-1)^k \approx \sum_{k \leq m} \binom{k-r-1}{k}
\]
\[
= \binom{-r+m}{m}
\]
\[
= (-1)^m\binom{r-1}{m},
\]
This formula gives us a partial sum of the \(r\)th row of Pascal’s triangle, provided
that the entries of the row have been given alternating signs. For instance, if
\(r = 5\) and \(m = 2\) the formula gives \(1 - 5 + 10 = 6 = (-1)^2\binom{3}{2}\).

Notice that if \(m \geq r\), (5.16) gives the alternating sum of the entire row,
and this sum is zero when \(r\) is a positive integer. We proved this before, when
we expanded \((1 - 1)^\tau\) by the binomial theorem; it’s interesting to know that
the partial sums of this expression can also be evaluated in closed form.

How about the simpler partial sum,
\[
\sum_{k \leq m} \binom{n}{k} \approx \binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{m}; \quad (5.17)
\]
surely if we can evaluate the corresponding sum with alternating signs, we
ought to be able to do this one? But no; there is no closed form for the partial
sum of a row of Pascal’s triangle. We can do columns—that’s (5.10)—but