Therefore
\[ v'(t) = c_1 \]
and
\[ v(t) = c_1 t + c_2, \]  \hspace{1cm} (10)
where \( c_1 \) and \( c_2 \) are arbitrary constants. Finally, substituting for \( v(t) \) in Eq. (6), we obtain
\[ y = c_1 t e^{-2t} + c_2 e^{-2t}. \]  \hspace{1cm} (11)
The second term on the right side of Eq. (11) corresponds to the original solution \( y_1(t) = \exp(-2t) \), but the first term arises from a second solution, namely \( y_2(t) = t \exp(-2t) \). These two solutions are obviously not proportional, but we can verify that they are linearly independent by calculating their Wronskian:
\[
W(y_1, y_2)(t) = \begin{vmatrix} e^{-2t} & te^{-2t} \\ -2e^{-2t} & (1-2t)e^{-2t} \end{vmatrix} = e^{-4t} - 2te^{-4t} + 2te^{-4t} = e^{-4t} \neq 0.
\]
Therefore
\[ y_1(t) = e^{-2t}, \quad y_2(t) = te^{-2t} \]  \hspace{1cm} (12)
form a fundamental set of solutions of Eq. (5), and the general solution of that equation is given by Eq. (11). Note that both \( y_1(t) \) and \( y_2(t) \) tend to zero as \( t \to \infty \); consequently, all solutions of Eq. (5) behave in this way. The graph of a typical solution is shown in Figure 3.5.1.

![Figure 3.5.1](image-url)

**FIGURE 3.5.1** A typical solution of \( y'' + 4y' + 4y = 0 \).

The procedure used in Example 1 can be extended to a general equation whose characteristic equation has repeated roots. That is, we assume that the coefficients in Eq. (1) satisfy \( b^2 - 4ac = 0 \), in which case
\[ y_1(t) = e^{-bt/2a} \]
is a solution. Then we assume that
\[ y = v(t)y_1(t) = v(t)e^{-bt/2a} \]  \hspace{1cm} (13)