so the roots are $r_1 = r_2 = 1/2$. Thus the general solution of the differential equation is

$$y = c_1 e^{t/2} + c_2 t e^{t/2}. \quad (22)$$

The first initial condition requires that $y(0) = c_1 = 2$.

To satisfy the second initial condition, we first differentiate Eq. (22) and then set $t = 0$. This gives

$$y'(0) = \frac{1}{2} c_1 + c_2 = \frac{1}{3},$$

so $c_2 = -2/3$. Thus, the solution of the initial value problem is

$$y = 2 e^{t/2} - \frac{2}{3} t e^{t/2}. \quad (23)$$

The graph of this solution is shown in Figure 3.5.2.

Let us now modify the initial value problem (21) by changing the initial slope; to be specific, let the second initial condition be $y'(0) = 2$. The solution of this modified problem is

$$y = 2 e^{t/2} + t e^{t/2}$$

and its graph is also shown in Figure 3.5.2. The graphs shown in this figure suggest that there is a critical initial slope, with a value between $\frac{1}{3}$ and 2, that separates solutions that grow positively from those that ultimately grow negatively. In Problem 16 you are asked to determine this critical initial slope.