The geometrical behavior of solutions is similar in this case to that when the roots are real and different. If the exponents are either positive or negative, then the magnitude of the solution grows or decays accordingly; the linear factor \( t \) has little influence. A decaying solution is shown in Figure 3.5.1 and growing solutions in Figure 3.5.2. However, if the repeated root is zero, then the differential equation is \( y'' = 0 \) and the general solution is a linear function of \( t \).

**Summary.** We can now summarize the results that we have obtained for second order linear homogeneous equations with constant coefficients,\[ ay'' + by' + cy = 0. \]Let \( r_1 \) and \( r_2 \) be the roots of the corresponding characteristic polynomial\[ ar^2 + br + c = 0. \]

If \( r_1 \) and \( r_2 \) are real but not equal, then the general solution of the differential equation (1) is\[ y = c_1e^{r_1t} + c_2e^{r_2t}. \]

If \( r_1 \) and \( r_2 \) are complex conjugates \( \lambda \pm i\mu \), then the general solution is\[ y = c_1e^{\lambda t}\cos \mu t + c_2e^{\lambda t}\sin \mu t. \]

If \( r_1 = r_2 \), then the general solution is\[ y = c_1e^{r_1t} + c_2te^{r_1t}. \]

**Reduction of Order.** It is worth noting that the procedure used earlier in this section for equations with constant coefficients is more generally applicable. Suppose we know one solution \( y_1(t) \), not everywhere zero, of\[ y'' + p(t)y' + q(t)y = 0. \]To find a second solution, let\[ y = v(t)y_1(t); \]
then\[ y' = v'(t)y_1(t) + v(t)y_1'(t) \]
and \[ y'' = v''(t)y_1(t) + 2v'(t)y_1'(t) + v(t)y_1''(t). \]
Substituting for \( y, y', \) and \( y'' \) in Eq. (27) and collecting terms, we find that\[ y_1v'' + (2y_1' + py_1)y' + (y_1'' + py_1' + qy_1)v = 0. \]Since \( y_1 \) is a solution of Eq. (27), the coefficient of \( v \) in Eq. (29) is zero, so that Eq. (29) becomes\[ y_1v'' + (2y_1' + py_1)v' = 0. \]Despite its appearance, Eq. (30) is actually a first order equation for the function \( v' \) and can be solved either as a first order linear equation or as a separable equation. Once \( v' \) has been found, then \( v \) is obtained by an integration. Finally, \( y \) is determined from