of its largest connected component to 1. As shown in Section 5.1, decreasing \( \kappa \) improves its time efficiency. On the other hand, it tends to a centralized behavior (17) by setting \( \epsilon \to 0^+ \): \( G \) becomes near-complete, thus resulting in \( \kappa \to K \).

Let
\[
\xi \triangleq \max_{n,v_{w,n},i,i'} \left| \left( \Sigma_{y_{w,n}} y_{w,n} \right)^{-1} \right|_{ii'}
\]
and \( \epsilon \triangleq 0.5 \log 1/(1 - (K^{1.5} L^{2.5} \kappa \xi \epsilon)^2) \). In the result below, we prove that the joint walk \( \hat{w} \) is guaranteed to achieve an entropy \( H[Z_{Y_{w,n}}] \) (i.e., by plugging \( \hat{w} \) into (18) that is not more than \( \epsilon \) from the maximum entropy \( H[Z_{Y_{w,n}}] \) achieved by joint walk \( w^* \) (17):

**Theorem 2 (Performance Guarantee)** If \( K^{1.5} L^{2.5} \kappa \xi \epsilon < 1 \), then
\[
H[Z_{Y_{w,n}}] - H[Z_{Y_{w,n}}] \leq \epsilon.
\]

Its proof is given in (Chen et al., 2012). The implication of Theorem 2 is that our partially decentralized active sensing algorithm can perform comparatively well (i.e., small \( \epsilon \)) under the following favorable environmental conditions: (a) the network of \( K \) sensors is not large, (b) length \( L \) of each sensor’s walk to be optimized is not long, (c) the largest subset of \( \kappa \) sensors being formed to coordinate their walks (i.e., largest connected component in \( G \)) is reasonably small, and (d) the minimum required correlation \( \epsilon \) between walks of adjacent sensors is kept low.

Algorithm 1 below outlines the key operations of our D²FAS algorithm to be run on each mobile sensor \( k \), as detailed previously in Sections 3 and 4:

**Algorithm 1:** D²FAS\((U, K, L, k, D_k, z_{D_k}, s_k)\)

```plaintext
while true
   /* Data fusion (Section 3) */
   Construct local summary by (6) & (7)
   Exchange local summary with every sensor \( i \neq k \)
   Construct global summary by (8) & (9)
   Predict measurements at unobserved road segments by (10) & (11)
   /* Active Sensing (Section 4) */
   Exchange \( \Phi_k \) with every sensor \( i \neq k \)
   Exchange adjacency vector \( a_k \) by (21) & (22)
   Exchange adjacency vector with every sensor \( i \neq k \)
   Construct adjacency matrix of coordination graph
   Find vertex set \( V_n \) of its residing connected component
   Compute maximum-entropy joint walk \( \hat{w}_{V_n} \) by (24)
   Execute walk \( \hat{w}_{V_n} \) and observe its road segments \( Y_{\hat{w}_{V_n}} \)
   Update local information \( D_k, z_{D_k}, \) and \( a_k \) by (16)
```

## 5 Time and Communication Overheads

In this section, the time and communication overheads of our D²FAS algorithm are analyzed and compared to that of centralized active sensing (17) coupled with the data fusion methods: Full GP (FGP) and SoD (Section 2).

### 5.1 Time Complexity

The data fusion component of D²FAS involves computing the local and global summaries and the predictive Gaussian distribution. To construct the local summary using (6) and (7), each sensor has to evaluate

\[
\sum_{D_k,D_k|U} \text{in } O(|U|^3 + |U|(|D|/K)^2)
\]
time and invert it in \( O(|D|/K)^3 \) time, after which the local summary is obtained in \( O(|U|^2|D|/K + |U|(|D|/K)^2) \) time. The global summary is computed in \( O(|U|^2K) \) by (8) and (9). Finally, the predictive Gaussian distribution is derived in \( O(|U|^3 + |U||Y|^2) \) time using (10) and (11). Supposing \(|Y| \leq |U|\) for simplicity, the time complexity of data fusion is then \( O(|D|/K)^3 + |U|^3 + |U|^2K) \).

Let the maximum-out-degree of \( G \) be denoted by \( \delta \). Then, each sensor has to consider \( \Delta \triangleq \delta^L \) possible walks of length \( L \). The active sensing component of D²FAS involves computing \( \Phi_k \) in \( O(\Delta L|U|^2) \) time, \( a_k \) in \( O(\Delta^2 L^2|U|/K) \) time, its residing connected component in \( O(\kappa^2) \) time, and the maximum-entropy joint walk by (11) and (24) with the following incurred time: The largest connected component of \( \kappa \) sensors in \( G \) has to consider \( \Delta^\epsilon \) possible joint walks. Note that \( \Sigma_{Y_{w,n}} Y_{w,n} = \text{diag}(\Sigma_{Y_{w,k}} Y_{w,k})_{k \in V_n} + \Sigma_{Y_{w,n}} U \Sigma_{Y_{w,n}} \) where \( \text{diag}(B) \) constructs a diagonal matrix by placing vector \( B \) on its diagonal. By exploiting \( \Phi_k \), the diagonal and latter matrix terms for all possible joint walks can be computed in \( O(\kappa \Delta (L|U|^2 + L^2|U|)) \) and \( O(\kappa^2 \Delta^2 L^2|U|) \) time, respectively. For each joint walk \( w_{V_n} \), evaluating the determinant of \( \Sigma_{Y_{w,n}} Y_{w,n} \) incurs \( O(\kappa L^3) \) time. Therefore, the time complexity of active sensing is \( O(\kappa \Delta L|U|^2 + \Delta^2 L^2|U|(K + \kappa^2) + \Delta^\epsilon (\kappa L^3)) \).

Hence, the time complexity of our D²FAS algorithm is \( O(|D|/K)^3 + |U|^2(|U| + K + \kappa \Delta L) + \Delta^2 L^2|U|(K + \kappa^2) + \Delta^\epsilon (\kappa L^3)) \). In contrast, the time incurred by centralized active sensing coupled with FGP and SoD are, respectively, \( O(|D|^3 + \Delta^K KL(|D|^2 + (KL)^2)) \) and \( O(|U|^3|D| + \Delta^K KL(|U|^2 + (KL)^2)) \). It can be observed that D²FAS can scale better with large \(|D|\) (i.e., number of observations) and \( K \) (i.e., number of sensors). The scalability of D²FAS vs. FGP and SoD will be further evaluated empirically in Section 6.

### 5.2 Communication Complexity

Let the communication overhead be defined as the size of each broadcast message. Recall from the data fusion component of D²FAS in Algorithm 1 that, in each iteration, each sensor broadcasts a \( O(|U|^2) \)-sized summary encapsulating its local observations, which is robust against communication failure. In contrast, FGP and SoD require each sensor to broadcast, in each iteration, a \( O(|D|/K) \)-sized message comprising exactly its local observations to handle communication failure. If the number of local observations grows to be larger in size than a local summary of predefined size, then the data fusion component of D²FAS is more scalable than FGP and SoD in terms of communication overhead. For the partially decentralized active sensing component of D²FAS, each sensor broadcasts \( O(\Delta L|U|) \)-sized \( \Phi_k \) and \( O(K) \)-sized \( a_k \) messages.