Find a particular solution of
\[ y'' - 3y' - 4y = 2 \sin t. \]  
(11)

By analogy with Example 1, let us first assume that \( Y(t) = A \sin t \), where \( A \) is a constant to be determined. On substituting in Eq. (11) and rearranging the terms, we obtain
\[ -5A \sin t - 3A \cos t = 2 \sin t, \]
or
\[ (2 + 5A) \sin t + 3A \cos t = 0. \]  
(12)
The functions \( \sin t \) and \( \cos t \) are linearly independent, so Eq. (12) can hold on an interval only if the coefficients \( 2 + 5A \) and \( 3A \) are both zero. These contradictory requirements mean that there is no choice of the constant \( A \) that makes Eq. (12) true for all \( t \). Thus we conclude that our assumption concerning \( Y(t) \) is inadequate. The appearance of the cosine term in Eq. (12) suggests that we modify our original assumption to include a cosine term in \( Y(t) \), that is,
\[ Y(t) = A \sin t + B \cos t, \]
where \( A \) and \( B \) are to be determined. Then
\[ Y'(t) = A \cos t - B \sin t, \quad Y''(t) = -A \sin t - B \cos t. \]
By substituting these expressions for \( y \), \( y' \), and \( y'' \) in Eq. (11) and collecting terms, we obtain
\[ (-A + 3B - 4A) \sin t + (-B - 3A - 4B) \cos t = 2 \sin t. \]  
(13)
To satisfy Eq. (13) we must match the coefficients of \( \sin t \) and \( \cos t \) on each side of the equation; thus \( A \) and \( B \) must satisfy the equations
\[ -5A + 3B = 2, \quad -3A - 5B = 0. \]
Hence \( A = -5/17 \) and \( B = 3/17 \), so a particular solution of Eq. (11) is
\[ Y(t) = -\frac{5}{17} \sin t + \frac{3}{17} \cos t. \]

The method illustrated in the preceding examples can also be used when the right side of the equation is a polynomial. Thus, to find a particular solution of
\[ y'' - 3y' - 4y = 4t^2 - 1, \]  
(14)
we initially assume that \( Y(t) \) is a polynomial of the same degree as the nonhomogeneous term, that is, \( Y(t) = At^2 + Bt + C \).

To summarize our conclusions up to this point: if the nonhomogeneous term \( g(t) \) in Eq. (1) is an exponential function \( e^{\alpha t} \), then assume that \( Y(t) \) is proportional to the same exponential function; if \( g(t) \) is \( \sin \beta t \) or \( \cos \beta t \), then assume that \( Y(t) \) is a linear combination of \( \sin \beta t \) and \( \cos \beta t \); if \( g(t) \) is a polynomial, then assume that \( Y(t) \) is a polynomial of like degree. The same principle extends to the case where \( g(t) \) is a product of any two, or all three, of these types of functions, as the next example illustrates.