Solutions of these three equations have been found in Examples 1, 2, and 3, respectively. Therefore a particular solution of Eq. (19) is their sum, namely,

\[ Y(t) = -\frac{1}{2}e^{2t} + \frac{3}{17} \cos t - \frac{5}{17} \sin t + \frac{10}{13}e^t \cos 2t + \frac{2}{13}e^t \sin 2t. \]

The procedure illustrated in these examples enables us to solve a fairly large class of problems in a reasonably efficient manner. However, there is one difficulty that sometimes occurs. The next example illustrates how it arises.

**Example 5**

Find a particular solution of

\[ y'' + 4y = 3 \cos 2t. \]  \hspace{1cm} (20)

Proceeding as in Example 2, we assume that \( Y(t) = A \cos 2t + B \sin 2t \). By substituting in Eq. (20), we then obtain

\[ (4A - 4A) \cos 2t + (4B - 4B) \sin 2t = 3 \cos 2t. \]  \hspace{1cm} (21)

Since the left side of Eq. (21) is zero, there is no choice of \( A \) and \( B \) that satisfies this equation. Therefore, there is no particular solution of Eq. (20) of the assumed form. The reason for this possibly unexpected result becomes clear if we solve the homogeneous equation

\[ y'' + 4y = 0 \]  \hspace{1cm} (22)

that corresponds to Eq. (20). A fundamental set of solutions of Eq. (22) is \( y_1(t) = \cos 2t \) and \( y_2(t) = \sin 2t \). Thus our assumed particular solution of Eq. (20) is actually a solution of the homogeneous equation (22); consequently, it cannot possibly be a solution of the nonhomogeneous equation (20).

To find a solution of Eq. (20) we must therefore consider functions of a somewhat different form. The simplest functions, other than \( \cos 2t \) and \( \sin 2t \) themselves, that when differentiated lead to \( \cos 2t \) and \( \sin 2t \) are \( t \cos 2t \) and \( t \sin 2t \). Hence we assume that \( Y(t) = At \cos 2t + Bt \sin 2t \). Then, upon calculating \( Y'(t) \) and \( Y''(t) \), substituting them into Eq. (20), and collecting terms, we find that

\[ -4A \sin 2t + 4B \cos 2t = 3 \cos 2t. \]

Therefore \( A = 0 \) and \( B = 3/4 \), so a particular solution of Eq. (20) is

\[ Y(t) = \frac{3}{4}t \sin 2t. \]

The fact that in some circumstances a purely oscillatory forcing term leads to a solution that includes a linear factor \( t \) as well as an oscillatory factor is important in some applications; see Section 3.9 for a further discussion.

The outcome of Example 5 suggests a modification of the principle stated previously: If the assumed form of the particular solution duplicates a solution of the corresponding homogeneous equation, then modify the assumed particular solution by multiplying it by \( t \). Occasionally, this modification will be insufficient to remove all duplication with the solutions of the homogeneous equation, in which case it is necessary to multiply