to game trees with chance nodes. It turns out that it can. The analysis for MIN and MAX
nodes is unchanged, but we can also prune chance nodes, using a bit of ingenuity. Consider
the chance node \( C \) in Figure 5.11 and what happens to its value as we examine and evaluate
its children. Is it possible to find an upper bound on the value of \( C \) before we have looked
at all its children? (Recall that this is what alpha—beta needs in order to prune a node and its
subtree.) At first sight, it might seem impossible because the value of \( C \) is the average of its
children’s values, and in order to compute the average of a set of numbers, we must look at
all the numbers. But if we put bounds on the possible values of the utility function, then we
can arrive at bounds for the average without looking at every number. For example, say that
all utility values are between \(-2\) and \(+2\); then the value of leaf nodes is bounded, and in turn
we can place an upper bound on the value of a chance node without looking at all its children.

An alternative is to do Monte Carlo simulation to evaluate a position. Start with
an alpha—beta (or other) search algorithm. From a start position, have the algorithm play
thousands of games against itself, using random dice rolls. In the case of backgammon, the
resulting win percentage has been shown to be a good approximation of the value of the
position, even if the algorithm has an imperfect heuristic and is searching only a few plies
(Tesauro, 1995). For games with dice, this type of simulation is called a rollout.

5.6 PARTIALLY OBSERVABLE GAMES

Chess has often been described as war in miniature, but it lacks at least one major character-
istic of real wars, namely, \textit{partial observability}. In the "fog of war," the existence and
disposition of enemy units is often unknown until revealed by direct contact. As a result,
warfare includes the use of scouts and spies to gather information and the use of concealment
and bluff to confuse the enemy. Partially observable games share these characteristics and
are thus qualitatively different from the games described in the preceding sections.

5.6.1 Kriegspiel: Partially observable chess

In deterministic partially observable games, uncertainty about the state of the board arises en-
tirely from lack of access to the choices made by the opponent. This class includes children’s
games such as Battleships (where each player’s ships are placed in locations hidden from the
opponent but do not move) and Stratego (where piece locations are known but piece types are
hidden). We will examine the game of Kriegspiel, a partially observable variant of chess in
which pieces can move but are completely invisible to the opponent.

The rules of Kriegspiel are as follows: White and Black each see a board containing
only their own pieces. A referee, who can see all the pieces, adjudicates the game and period-
ically makes announcements that are heard by both players. On his turn, White proposes to
the referee any move that would be legal if there were no black pieces. If the move is in fact
not legal (because of the black pieces), the referee announces "illegal." In this case, White
may keep proposing moves until a legal one is found—and learns more about the location of
Black’s pieces in the process. Once a legal move is proposed, the referee announces one or
more of the following: "Capture on square X" if there is a capture, and "Check by D" if the black king is in check, where D is the direction of the check, and can be one of "Knight," "Rank," "File," "Long diagonal," or "Short diagonal." (In case of discovered check, the referee may make two "Check" announcements.) If Black is checkmated or stalemated, the referee says so; otherwise, it is Black's turn to move.

Kriegspiel may seem terrifyingly impossible, but humans manage it quite well and computer programs are beginning to catch up. It helps to recall the notion of a belief state as defined in Section 4.4 and illustrated in Figure 4.14—the set of all logically possible board states given the complete history of percepts to date. Initially, White's belief state is a singleton because Black's pieces haven't moved yet. After White makes a move and Black responds, White's belief state contains 20 positions because Black has 20 replies to any White move. Keeping track of the belief state as the game progresses is exactly the problem of state estimation, for which the update step is given in Equation (4.6). We can map Kriegspiel state estimation directly onto the partially observable, nondeterministic framework of Section 4.4 if we consider the opponent as the source of nondeterminism; that is, the RESULTS of White's move are composed from the (predictable) outcome of White's own move and the unpredictable outcome given by Black's reply. 3

Given a current belief state, White may ask, "Can I win the game?" For a partially observable game, the notion of a strategy is altered; instead of specifying a move to make for each possible move the opponent might make, we need a move for every possible percept sequence that might be received. For Kriegspiel, a winning strategy, or guaranteed checkmate, is one that, for each possible percept sequence, leads to an actual checkmate for every possible board state in the current belief state, regardless of how the opponent moves. With this definition, the opponent's belief state is irrelevant—the strategy has to work even if the opponent can see all the pieces. This greatly simplifies the computation. Figure 5.13 shows part of a guaranteed checkmate for the KRK (king and rook against king) endgame. In this case, Black has just one piece (the king), so a belief state for White can be shown in a single board by marking each possible position of the Black king.

The general AND-OR search algorithm can be applied to the belief-state space to find guaranteed checkmates, just as in Section 4.4. The incremental belief-state algorithm mentioned in that section often finds midgame checkmates up to depth 9—probably well beyond the abilities of human players.

In addition to guaranteed checkmates, Kriegspiel admits an entirely new concept that makes no sense in fully observable games: probabilistic checkmate. Such checkmates are still required to work in every board state in the belief state; they are probabilistic with respect to randomization of the winning player's moves. To get the basic idea, consider the problem of finding a lone black king using just the white king. Simply by moving randomly, the white king will eventually bump into the black king even if the latter tries to avoid this fate. Since Black cannot keep guessing the right evasive moves indefinitely. In the terminology of probability theory, detection occurs with probability 1. The KBNK endgame—king, bishop

---

3 Sometimes, the belief state will become too large to represent just as a list of board states, but we will ignore this issue for now; Chapters 7 and 8 suggest methods for compactly representing very large belief states.