it follows that $F_d(t)$ is again given by Eq. (5). Thus, regardless of the direction of motion of the mass, the damping force is always expressed by Eq. (5).

The damping force may be rather complicated and the assumption that it is modeled adequately by Eq. (5) may be open to question. Some dashpots do behave as Eq. (5) states, and if the other sources of dissipation are small, it may be possible to neglect them altogether, or to adjust the damping constant $\gamma$ to approximate them. An important benefit of the assumption (5) is that it leads to a linear (rather than a nonlinear) differential equation. In turn, this means that a thorough analysis of the system is straightforward, as we will show in this section and the next.

4. An applied external force $F(t)$ is directed downward or upward as $F(t)$ is positive or negative. This could be a force due to the motion of the mount to which the spring is attached, or it could be a force applied directly to the mass. Often the external force is periodic.

Taking account of these forces, we can now rewrite Newton’s law (3) as

$$mu''(t) = mg + F_s(t) + F_d(t) + F(t) = mg - k[L + u(t)] - \gamma u'(t) + F(t).$$

(6)

Since $mg - kL = 0$ by Eq. (2), it follows that the equation of motion of the mass is

$$mu''(t) + \gamma u'(t) + ku(t) = F(t).$$

(7)

where the constants $m, \gamma,$ and $k$ are positive. Note that Eq. (7) has the same form as Eq. (1).

It is important to understand that Eq. (7) is only an approximate equation for the displacement $u(t)$. In particular, both Eqs. (4) and (5) should be viewed as approximations for the spring force and the damping force, respectively. In our derivation we have also neglected the mass of the spring in comparison with the mass of the attached body.

The complete formulation of the vibration problem requires that we specify two initial conditions, namely, the initial position $u_0$ and the initial velocity $v_0$ of the mass:

$$u(0) = u_0, \quad u'(0) = v_0.$$  

(8)

It follows from Theorem 3.2.1 that these conditions give a mathematical problem that has a unique solution. This is consistent with our physical intuition that if the mass is set in motion with a given initial displacement and velocity, then its position will be determined uniquely at all future times. The position of the mass is given (approximately) by the solution of Eq. (7) subject to the prescribed initial conditions (8).

A mass weighing 4 lb stretches a spring 2 in. Suppose that the mass is displaced an additional 6 in. in the positive direction and then released. The mass is in a medium that exerts a viscous resistance of 6 lb when the mass has a velocity of 3 ft/sec. Under the assumptions discussed in this section, formulate the initial value problem that governs the motion of the mass.

The required initial value problem consists of the differential equation (7) and initial conditions (8), so our task is to determine the various constants that appear in these equations. The first step is to choose the units of measurement. Based on the statement of the problem, it is natural to use the English rather than the metric system of units. The