The assumption that the join of traders’ signal structure consists only of singleton sets is not without loss of generality. If the signal structure is known, however, the market designer can treat elements of the join as states of the world, identify the correct element of the join by running the market with a single security, then apply the prior to that element to learn the likelihood of each state as if he knew all the traders’ private signals. If the prior is unknown this distribution can also be solicited from any single trader using a scoring rule.

5.4 Constrained Design

A single security acting as a summary statistic for an entire market is unlikely to be considered natural by any criterion. Real markets, like those on Intrade, use multiple securities. Instead of imposing our own definition of natural, in this section we consider adding a design constraint that the market’s securities must be picked from a predefined set. The market designer is then challenged to find the fewest securities from this set that are informative on the events of interest with respect to the given signal structure. We call this the INFORMATIVE SET optimization problem. If the set of predefined securities is empty or has no informative subset then the problem is simply infeasible, so we assume there exists at least one such subset.

Demonstrating INFORMATIVE SET is hard would not be very interesting if exotic and unnatural securities were required for the proof. One commonly used class of securities are event securities which pay $1 if an event occurs and $0 otherwise. The corresponding optimization problem is INFORMATIVE EVENT SET, a restriction of INFORMATIVE SET, and even solving this restricted version of the problem is NP-hard.

**Theorem 7.** INFORMATIVE EVENT SET is NP-hard.

This immediately implies that the more general INFORMATIVE SET problem is also hard.

**Corollary 1.** INFORMATIVE SET is NP-hard.

The proof appears in the appendix and demonstrates a one-to-one correspondence between set cover instances and a minimal informative set of securities for a single fully informed trader.

The complexity of these problems suggests that while knowledge of the traders’ signal structure allows for better designs, a perfect design will be intractable to compute or require additional assumptions about the relationship between traders’ signal structure and the set of possible securities. Practically we can only ever hope to offer better (but not perfect) designs that extract more information from traders than current markets do. These results confirm we will always have to settle for some degree of error in our designs even if the traders’ signal structure could be perfectly observed.

6 CONCLUSION

We developed a formal framework for the design of informative prediction markets. These markets reveal the posterior probabilities of a set of events of interest as accurately as if the traders had directly revealed their information, a commonly cited goal of prediction markets. These markets require that traders have an incentive to be accurate, that they can aggregate their information, and that the market designer can use this information to infer the likelihood of the events of interest.

Ideally informative markets would use a few natural securities. Complete markets, usually too large to be used in practice, are, however, the only markets to always be informative, regardless of the traders’ signal structure. When the signal structure is known, a single security can be informative, but this security may appear strange and unintuitive to traders. Finding the smallest informative set of natural securities is computationally hard in general.

Real-world prediction markets do typically offer small numbers of simple and natural securities, and have been shown to aggregate information effectively in practice. This is not in contrast to our results, which only consider whether a market reveals all of the traders’ private information. Our results demonstrate, however, the importance of security design and suggest that better designs that extract more of the traders’ information are possible.

We hope this paper will allow future research on partial aggregation, like that which occurs in practice. Future work might also consider the effect of alternative communication channels outside the market’s securities, like comments on Intrade, that allow traders to explain the reasoning behind their predictions. The development of this line of research is crucial to understanding why prediction markets work and how to make them work better.