is valid for every \( u \in \mathcal{M}, j = 1, \ldots, N \), where the constant \( c_2 \) is independent of \( \delta \).
Choosing \( \delta, \varepsilon' \) such that \( c_2 c_0 \delta^{(\gamma-l_1)-(\beta-l)} + c_1(\delta) \varepsilon' < \varepsilon \), we obtain (6.2.1). Hence \( \mathcal{M} \) is precompact in \( V^{l_1}_{2,\gamma}(\mathcal{G}) \).

Similarly it can be proved that the imbeddings
\[
V^{l-1/2}_{2,\beta}(\partial \mathcal{G}) \subset V^{l_1-1/2}_{2,\gamma}(\partial \mathcal{G}), \quad \tilde{V}^{l,k}_{2,\beta}(\mathcal{G}) \subset \tilde{V}^{l_1,k}_{2,\gamma}(\mathcal{G})
\]
are continuous if \( l \geq l_1, \beta - l \leq \gamma - l_1 \) for \( \tau = 1, \ldots, d \), and compact if \( l > l_1, \beta - l < \gamma - l_1 \). Moreover, the space \( \tilde{V}^{l,k}_{2,\beta}(\mathcal{G}) \) is dense in \( \tilde{V}^{l_1,k}_{2,\gamma}(\mathcal{G}) \) for \( l \geq l_1, \beta - l \leq \gamma - l_1, \tau = 1, \ldots, d \).

### 6.2.2. Formulation of the problem

We consider the boundary value problem
\begin{align*}
L(x, \partial_x)u &= f \quad \text{in } \mathcal{G}, \\
B(x, \partial_x)u + C(x, \partial_x)u &= g \quad \text{on } \partial \mathcal{G} \setminus \mathcal{S},
\end{align*}
where \( L \) is a differential operator of order \( 2m \), \( B \) is a vector of differential operators \( B_k \), \( \text{ord } B_k \leq \mu_k \), and \( C \) is a matrix of tangential differential operators \( C_{k,j} \) on \( \partial \mathcal{G} \setminus \mathcal{S} \), \( \text{ord } C_{k,j} \leq \mu_k + \gamma_j \). The coefficients of \( L, B_k \), and \( C_{k,j} \) are assumed to be infinitely differentiable in \( \mathcal{G} \setminus \mathcal{S} \). Throughout this chapter, we suppose that the orders (with respect to differentiation) of the operators \( B_k \) are less than \( 2m \). Then the vector \( B \) admits the representation
\[
Bu|_{\partial \mathcal{G} \setminus \mathcal{S}} = Q(x, \partial_x) \cdot \mathcal{D}u|_{\partial \mathcal{G} \setminus \mathcal{S}},
\]
where \( Q \) is a \((m+J)\times 2m\)-matrix of tangential differential operators \( Q_{k,j} \), \( \text{ord } Q_{k,j} \leq \mu_k - j + 1 \), \( Q_{k,j} \equiv 0 \) if \( \mu_k - j + 1 < 0 \).

Furthermore, we suppose that the following condition analogous to the stabilization condition in Section 5.5 is satisfied for the coefficients in a neighbourhood of every conical point \( x^{(\tau)} \).

**DEFINITION 6.2.1.** The operator
\[
P(x, \partial_x) = \sum_{|\alpha| \leq k} p_{\alpha}(x) \partial_x^\alpha
\]
is said to be an **admissible operator** of order \( k \) in a neighbourhood of the conical point \( x^{(\tau)} \) if the coefficients \( p_{\alpha} \) have the form
\[
p_{\alpha}(x) = r^{|\alpha|-k} p_{\alpha}^{(0)}(\omega, r)
\]
in this neighbourhood, where \( p_{\alpha}^{(0)} \) is infinitely differentiable in \( \overline{\Omega}_\tau \times \mathbb{R}_+ \), continuous in \( \overline{\Omega}_\tau \times \mathbb{R}_+ \), and
\[
(r \partial_r)^j \partial_\omega^\gamma (p_{\alpha}^{(0)}(\omega, r) - p_{\alpha}^{(0)}(\omega, 0)) \rightarrow 0 \quad \text{as } r \rightarrow 0
\]
uniformly with respect to \( \omega \in \overline{\Omega}_\tau \). Here \( \omega \) are coordinates on the unit sphere with center in \( x^{(\tau)} \) and \( r = |x - x^{(\tau)}| \) denotes the distance to \( x^{(\tau)} \).

If \( P(x, \partial_x) \) is an admissible operator of order \( k \) with the coefficients (6.2.6), then the operator
\[
P^{(\tau)}(x, \partial_x) = \sum_{|\alpha| \leq k} r^{|\alpha|-k} p_{\alpha}^{(0)}(\omega, 0) \partial_x^\alpha
\]
is called the **leading part of \( P \) at the point \( x^{(\tau)} \).**
Analogously, admissible tangential operators on $\partial G \setminus S$ and their leading parts at the conical point $x^{(\tau)}$ are defined.

Note that the order (with respect to differentiation) of an admissible operator of order $k$ can be strictly less than $k$.

The leading part $P^{(\tau)}(x, \partial_x)$ of the admissible operator (6.2.5) is considered as a differential operator in the cone $K_\tau$. Since $r^{|\alpha|} \partial^\alpha_x \omega$ can be written in the form $\sum_{j \leq |\alpha|} p_j(\omega, \partial_x, (r \partial_x)^\tau)$, the leading part $P^{(\tau)}$ is a model operator in $K_\tau$.

Clearly, every model operator is admissible. Moreover, e.g., every operator (6.2.5) with coefficients $p_\alpha \in C^\infty(\tilde{G})$ is admissible in a neighbourhood of each conical point. In this case the leading part at the point $x^{(\tau)}$ is equal to

$$P^{(\tau)}(x^{(\tau)}, \partial_x) = \sum_{|\alpha|=k} a_\alpha(x^{(\tau)}) \partial_x^\alpha.$$  

It can be easily verified that every differential operator of order $k \leq l$ with smooth coefficients in $\tilde{G} \setminus S$ which is admissible in a neighbourhood of every conical point $x^{(\tau)}$ continuously maps the space $V_{2, \beta}^l(\tilde{G})$ into $V_{2, \beta}^{l-k}(\tilde{G})$. Furthermore, condition (6.2.7) ensures the validity of the following assertion (cf. Lemma 5.5.1).

**Lemma 6.2.2.** Let $P$ be an admissible operator of order $k$ in a neighbourhood of the conical point $x^{(\tau)}$ and let $\varepsilon$ be a sufficiently small positive real number. Then there exists a constant $c_\varepsilon$ such that

$$\| (P(x, \partial_x) - P^{(\tau)}(x, \partial_x)) u \|_{V_{2, \beta, \varepsilon}^{l-k}(K_\tau)} \leq c_\varepsilon \| u \|_{V_{2, \beta, \varepsilon}^l(K_\tau)}$$

for every $u \in V_{2, \beta}^l(K_\tau)$ equal to zero outside the ball $|x - x^{(\tau)}| < \varepsilon$ (extending $u$ by zero outside the ball $|x - x^{(\tau)}| < \varepsilon$, the function $u$ can be simultaneously considered as a function in $G$ and $K_\tau$). The factor $c_\varepsilon$ tends to zero as $\varepsilon \to 0$.

We suppose in the sequel that $L, B_k$, and $C_{k, j}$ are admissible operators of order $2m, \mu_k$, and $\mu_k + \tau_j$, respectively, in a neighbourhood of each conical point $x^{(\tau)}$. Outside $S$ the coefficients of $L, B_k, C_{k, j}$ are assumed to be smooth. Furthermore, we suppose that the boundary value problem (6.2.2), (6.2.3) is elliptic, i.e., condition (i) in Definition 3.1.2 is satisfied for each $x^{(0)} \in \tilde{G} \setminus S$ and condition (ii) is satisfied for each $x^{(0)} \in \partial \tilde{G} \setminus S$, where the numbers $\mu_k$ and $\tau_j$ are the same as above. Obviously, the operator $A$ of the boundary value problem (6.2.2), (6.2.3) continuously maps the space

$$V_{2, \beta}^{l}(\tilde{G}) \times V_{2, \beta}^{l+\tau_1 - 1/2}(\partial \tilde{G})$$

into

$$V_{2, \beta}^{l-2m}(\tilde{G}) \times V_{2, \beta}^{l+\mu_1 - 1/2}(\partial \tilde{G}).$$

for $l \geq 2m$. Here $V_{2, \beta}^{l+\tau_j - 1/2}(\partial \tilde{G}), V_{2, \beta}^{l+\mu_k - 1/2}(\partial \tilde{G})$ denote the products of the spaces $V_{2, \beta}^{l+\tau_j - 1/2}(\partial \tilde{G}), j = 1, \ldots, J$, and $V_{2, \beta}^{l+\mu_k - 1/2}(\partial \tilde{G}), k = 1, \ldots, m + J$, respectively.

Let $(u, \tilde{u})$ be a solution of the boundary value problem (6.2.2), (6.2.3). We suppose that the support of $(u, \tilde{u})$ is contained in the neighbourhood $U_\tau$ of $x^{(\tau)}$. Passing to the coordinates $\omega, t$, where $t = \log r = \log |x - x^{(\tau)}|$ and $\omega$ are coordinates on the unit sphere $|x - x^{(\tau)}| = 1$, the pair $(u, \tilde{u})$ can be considered as a solution of