PROBLEMS

In each of Problems 1 through 4 determine \( \omega_0, R, \) and \( \delta \) so as to write the given expression in the form \( u = R \cos(\omega t - \delta). \)

1. \( u = 3 \cos 2t + 4 \sin 2t \)
2. \( u = -\cos t + \sqrt{3} \sin t \)
3. \( u = 4 \cos 3t - 2 \sin 3t \)
4. \( u = -2 \cos \pi t - 3 \sin \pi t \)

5. A mass weighing 2 lb stretches a spring 6 in. If the mass is pulled down an additional 3 in. and then released, and if there is no damping, determine the position \( u \) of the mass at any time \( t \). Plot \( u \) versus \( t \). Find the frequency, period, and amplitude of the motion.

6. A mass of 100 g stretches a spring 5 cm. If the mass is set in motion from its equilibrium position with a downward velocity of 10 cm/sec, and if there is no damping, determine the position \( u \) of the mass at any time \( t \). When does the mass first return to its equilibrium position?

7. A mass weighing 3 lb stretches a spring 3 in. If the mass is pushed upward, contracting the spring a distance of 1 in., and then set in motion with a downward velocity of 2 ft/sec, and if there is no damping, find the position \( u \) of the mass at any time \( t \). Determine the frequency, period, amplitude, and phase of the motion.

8. A series circuit has a capacitor of \( 0.25 \times 10^{-6} \) farad and an inductor of 1 henry. If the initial charge on the capacitor is \( 10^{-6} \) coulomb and there is no initial current, find the charge \( Q \) on the capacitor at any time \( t \).

9. A mass of 20 g stretches a spring 5 cm. Suppose that the mass is also attached to a viscous damper with a damping constant of 400 dyne-sec/cm. If the mass is pulled down an additional 2 cm and then released, find its position \( u \) at any time \( t \). Plot \( u \) versus \( t \). Determine the quasi frequency and the quasi period. Determine the ratio of the quasi period to the period of the corresponding undamped motion. Also find the time \( \tau \) such that \(|u(t)| < 0.05\) cm for all \( t > \tau \).

10. A mass weighing 16 lb stretches a spring 3 in. The mass is attached to a viscous damper with a damping constant of 2 lb-sec/ft. If the mass is set in motion from its equilibrium position with a downward velocity of 3 in./sec, find its position \( u \) at any time \( t \). Plot \( u \) versus \( t \). Determine when the mass first returns to its equilibrium position. Also find the time \( \tau \) such that \(|u(t)| < 0.01\) in. for all \( t > \tau \).

11. A spring is stretched 10 cm by a force of 3 newtons. A mass of 2 kg is hung from the spring and is also attached to a viscous damper that exerts a force of 3 newtons when the velocity of the mass is 5 m/sec. If the mass is pulled down 5 cm below its equilibrium position and given an initial downward velocity of 10 cm/sec, determine its position \( u \) at any time \( t \). Find the quasi frequency \( \mu \) and the ratio of \( \mu \) to the natural frequency of the corresponding undamped motion.

12. A series circuit has a capacitor of \( 10^{-5} \) farad, a resistor of \( 3 \times 10^2 \) ohms, and an inductor of 0.2 henry. The initial charge on the capacitor is \( 10^{-6} \) coulomb and there is no initial current. Find the charge \( Q \) on the capacitor at any time \( t \).

13. A certain vibrating system satisfies the equation \( u'' + \gamma u' + u = 0 \). Find the value of the damping coefficient \( \gamma \) for which the quasi period of the damped motion is 50% greater than the period of the corresponding undamped motion.

14. Show that the period of motion of an undamped vibration of a mass hanging from a vertical spring is \( 2\pi \sqrt{\frac{L}{g}} \), where \( L \) is the elongation of the spring due to the mass and \( g \) is the acceleration due to gravity.

15. Show that the solution of the initial value problem

\[
m u'' + \gamma u' + ku = 0, \quad u(t_0) = u_0, \quad u'(t_0) = u_0'
\]

can be expressed as the sum \( u = v + w \), where \( v \) satisfies the initial conditions \( v(t_0) = u_0, \quad v'(t_0) = 0 \), \( w \) satisfies the initial conditions \( w(t_0) = 0, \quad w'(t_0) = u_0' \), and both \( v \) and \( w \) satisfy the same differential equation as \( u \). This is another instance of superposing solutions of simpler problems to obtain the solution of a more general problem.