16. Show that \( A \cos \omega_0 t + B \sin \omega_0 t \) can be written in the form \( r \sin(\omega_0 t - \theta) \). Determine \( r \) and \( \theta \) in terms of \( A \) and \( B \). If \( R \cos(\omega_0 t - \delta) = r \sin(\omega_0 t - \theta) \), determine the relationship among \( R \), \( r \), \( \delta \), and \( \theta \).

17. A mass weighing 8 lb stretches a spring 1.5 in. The mass is also attached to a damper with coefficient \( \gamma \). Determine the value of \( \gamma \) for which the system is critically damped; be sure to give the units for \( \gamma \).

18. If a series circuit has a capacitor of 25 farad and an inductor of \( L = 0.2 \) henry, find the resistance \( R \) so that the circuit is critically damped.

19. Assume that the system described by the equation \( mu'' + \gamma u' + ku = 0 \) is either critically damped or overdamped. Show that the mass can pass through the equilibrium position at most once, regardless of the initial conditions.

**Hint:** Determine all possible values of \( t \) for which \( u = 0 \).

20. Assume that the system described by the equation \( mu'' + \gamma u' + ku = 0 \) is critically damped and the initial conditions are \( u(0) = u_0 \), \( u'(0) = v_0 \). If \( v_0 = 0 \), show that \( u \to 0 \) as \( t \to \infty \), but that \( u \) is never zero. If \( u_0 \) is positive, determine a condition on \( v_0 \) that will assure that the mass passes through its equilibrium position after it is released.

21. **Logarithmic Decrement.** (a) For the damped oscillation described by Eq. (26), show that the time between successive maxima is \( T_d = 2\pi/\mu \).

(b) Show that the ratio of the displacements at two successive maxima is given by \( \exp(\gamma T_d/2m) \). Observe that this ratio does not depend on which pair of maxima is chosen. The natural logarithm of this ratio is called the logarithmic decrement and is denoted by \( \Delta \).

(c) Show that \( \Delta = \pi \gamma / m \mu \). Since \( m \), \( \mu \), and \( \Delta \) are quantities that can be measured easily for a mechanical system, this result provides a convenient and practical method for determining the damping constant of the system, which is more difficult to measure directly. In particular, for the motion of a vibrating mass in a viscous fluid the damping constant depends on the viscosity of the fluid; for simple geometric shapes the form of this dependence is known, and the preceding relation allows the determination of the viscosity experimentally. This is one of the most accurate ways of determining the viscosity of a gas at high pressure.

22. Referring to Problem 21, find the logarithmic decrement of the system in Problem 10.

23. For the system in Problem 17 suppose that \( \Delta = 3 \) and \( T_d = 0.3 \) sec. Referring to Problem 21, determine the value of the damping coefficient \( \gamma \).

24. The position of a certain spring–mass system satisfies the initial value problem

\[
\frac{1}{2} u'' + ku = 0, \quad u(0) = 2, \quad u'(0) = v.
\]

If the period and amplitude of the resulting motion are observed to be \( \pi \) and 3, respectively, determine the values of \( k \) and \( v \).

25. Consider the initial value problem

\[
u'' + \gamma u' + u = 0, \quad u(0) = 2, \quad u'(0) = 0.
\]

We wish to explore how long a time interval is required for the solution to become "negligible" and how this interval depends on the damping coefficient \( \gamma \). To be more precise, let us seek the time \( \tau \) such that \( |u(t)| < 0.01 \) for all \( t > \tau \). Note that critical damping for this problem occurs for \( \gamma = 2 \).

(a) Let \( \gamma = 0.25 \) and determine \( \tau \), or at least estimate it fairly accurately from a plot of the solution.

(b) Repeat part (a) for several other values of \( \gamma \) in the interval \( 0 < \gamma < 1.5 \). Note that \( \tau \) steadily decreases as \( \gamma \) increases for \( \gamma \) in this range.

(c) Obtain a graph of \( \tau \) versus \( \gamma \) by plotting the pairs of values found in parts (a) and (b). Is the graph a smooth curve?

(d) Repeat part (b) for values of \( \gamma \) between 1.5 and 2. Show that \( \tau \) continues to decrease until \( \gamma \) reaches a certain critical value \( \gamma_0 \), after which \( \tau \) increases. Find \( \gamma_0 \) and the corresponding minimum value of \( \tau \) to two decimal places.