More precisely, we find that \( \Delta = \sqrt{145}/4 \cong 3.0104 \), so \( R = F_0/\Delta \cong 0.9965 \). Further, \( \cos \delta = -3/\Delta \cong -0.9965 \) and \( \sin \delta = 1/4\Delta \cong 0.08305 \), so that \( \delta \cong 3.0585 \). Thus the calculated values of \( R \) and \( \delta \) are close to the values estimated from the graph.

PROBLEMS

In each of Problems 1 through 4 write the given expression as a product of two trigonometric functions of different frequencies.

1. \( \cos 9t - \cos 7t \)
2. \( \sin 7t - \sin 6t \)
3. \( \cos \pi t + \cos 2\pi t \)
4. \( \sin 3t + \sin 4t \)

5. A mass weighing 4 lb stretches a spring 1.5 in. The mass is displaced 2 in. in the positive direction from its equilibrium position and released with no initial velocity. Assuming that there is no damping and that the mass is acted on by an external force of 2 cos 2t lb, formulate the initial value problem describing the motion of the mass.

6. A mass of 5 kg stretches a spring 10 cm. The mass is acted on by an external force of 10 sin(t/2) N (newtons) and moves in a medium that imparts a viscous force of 2 N when the speed of the mass is 4 cm/sec. If the mass is set in motion from its equilibrium position with an initial velocity of 3 cm/sec, formulate the initial value problem describing the motion of the mass.

7. (a) Find the solution of Problem 5.
   (b) Plot the graph of the solution.
   (c) If the given external force is replaced by a force 4 sin \( \omega t \) of frequency \( \omega \), find the value of \( \omega \) for which resonance occurs.

8. (a) Find the solution of the initial value problem in Problem 6.
   (b) Identify the transient and steady-state parts of the solution.
   (c) Plot the graph of the steady-state solution.
   (d) If the given external force is replaced by a force 2 cos \( \omega t \) of frequency \( \omega \), find the value of \( \omega \) for which the amplitude of the forced response is maximum.