Higher Order Linear Equations

The theoretical structure and methods of solution developed in the preceding chapter for second order linear equations extend directly to linear equations of third and higher order. In this chapter we briefly review this generalization, taking particular note of those instances where new phenomena may appear, due to the greater variety of situations that can occur for equations of higher order.

4.1 General Theory of $n$th Order Linear Equations

An $n$th order linear differential equation is an equation of the form

$$P_0(t) \frac{d^n y}{dt^n} + P_1(t) \frac{d^{n-1} y}{dt^{n-1}} + \cdots + P_{n-1}(t) \frac{dy}{dt} + P_n(t)y = G(t). \quad (1)$$

We assume that the functions $P_0, \ldots, P_n$ and $G$ are continuous real-valued functions on some interval $I: \alpha < t < \beta$, and that $P_0$ is nowhere zero in this interval. Then, dividing Eq. (1) by $P_0(t)$, we obtain

$$L[y] = \frac{d^n y}{dt^n} + P_1(t) \frac{d^{n-1} y}{dt^{n-1}} + \cdots + P_{n-1}(t) \frac{dy}{dt} + P_n(t)y = g(t). \quad (2)$$

The linear differential operator $L$ of order $n$ defined by Eq. (2) is similar to the second order operator introduced in Chapter 3. The mathematical theory associated with Eq. (2) is completely analogous to that for the second order linear equation; for this reason we