In each of Problems 7 through 10 follow the procedure illustrated in Example 4 to determine the indicated roots of the given complex number.

7. $1^{1/3}$
8. $(1 - i)^{1/2}$
9. $i^{1/4}$
10. $[2(\cos \pi/3 + i \sin \pi/3)]^{1/2}$

In each of Problems 11 through 28 find the general solution of the given differential equation.

11. $y'''' - y'' - y' + y = 0$
12. $y'''' - 3y'' - 3y' + y = 0$
13. $2y'''' - 4y'' - 2y' + 4y = 0$
14. $y'' - 4y'''' + 4y'' = 0$
15. $y'''' + y = 0$
16. $y'' - 5y'''' + 4y'' = 0$
17. $y'' - 3y'''' - 3y' - y = 0$
18. $y'''' - y'' = 0$
19. $y'' - 3y'''' + 3y'' - 3y' + 2y = 0$
20. $y'' - 8y'''' + 3y'' = 0$
21. $y'''' + 8y'''' + 16y = 0$
22. $y'' + 2y'''' + y = 0$
23. $y'''' + 5y'''' + 3y' + y = 0$
24. $y'' + 5y'''' + 6y' + 2y = 0$
25. $18y'''' + 21y'''' + 14y' + 4y = 0$
26. $y'' - 7y'''' + 6y'''' + 30y' - 36y = 0$
27. $12y'''' + 31y'''' + 75y'''' + 37y' + 5y = 0$
28. $y'''' + 6y'''' + 17y'''' + 22y' + 14y = 0$

In each of Problems 29 through 36 find the solution of the given initial value problem and plot its graph. How does the solution behave as $t \to \infty$?

29. $y'''' + y' = 0; \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 2$
30. $y'''' + y = 0; \quad y(0) = 0, \quad y'(0) = 0, \quad y''(0) = -1, \quad y''''(0) = 0$
31. $y'' - 4y'''' + 4y'' = 0; \quad y(1) = -1, \quad y'(1) = 2, \quad y''(1) = 0, \quad y''''(1) = 0$
32. $y'''' - y'' + y' - y = 0; \quad y(0) = 2, \quad y'(0) = -1, \quad y''(0) = -2$
33. $2y'''' - 9y'''' + 4y'' + 4y = 0; \quad y(0) = 2, \quad y'(0) = 0, \quad y''(0) = 2, \quad y''''(0) = 0$
34. $4y'''' + y' + 5y = 0; \quad y(0) = 2, \quad y'(0) = 1, \quad y''(0) = -1$
35. $6y'''' + 5y'' + y' = 0; \quad y(0) = -2, \quad y'(0) = 2, \quad y''(0) = 0$
36. $y'' + 6y'''' + 17y'' + 22y' + 14y = 0; \quad y(0) = 1, \quad y'(0) = -2, \quad y''(0) = 0, \quad y''''(0) = 3$
37. Show that the general solution of $y'' - y = 0$ can be written as

$$y = c_1 \cos t + c_2 \sin t + c_3 \cosh t + c_4 \sinh t.$$ 

Determine the solution satisfying the initial conditions $y(0) = 0, y'(0) = 0, y''(0) = 1, y'''(0) = 1$. Why is it convenient to use the solutions $\cosh t$ and $\sinh t$ rather than $e^t$ and $e^{-t}$?

38. Consider the equation $y'''' - y = 0$.
   (a) Use Abel’s formula [Problem 20(d) of Section 4.1] to find the Wronskian of a fundamental set of solutions of the given equation.
   (b) Determine the Wronskian of the solutions $e^t, e^{-t}, \cos t$, and $\sin t$.
   (c) Determine the Wronskian of the solutions $\cosh t, \sinh t, \cos t$, and $\sin t$.

39. Consider the spring–mass system, shown in Figure 4.2.4 consisting of two unit masses suspended from springs with spring constants 3 and 2, respectively. Assume that there is no damping in the system.
   (a) Show that the displacements $u_1$ and $u_2$ of the masses from their respective equilibrium positions satisfy the equations

$$u_1'' + 5u_1 = 2u_2, \quad u_2'' + 2u_2 = 2u_1.$$  

   (b) Solve the first of Eqs. (i) for $u_2$ and substitute into the second equation, thereby obtaining the following fourth order equation for $u_1$:

$$u_1'''' + 7u_1'' + 6u_1 = 0.$$  

Find the general solution of Eq. (ii).