converges for all \( x \), we say that \( \rho \) is infinite. If \( \rho > 0 \), then the interval \( |x - x_0| < \rho \) is called the interval of convergence; it is indicated by the hatched lines in Figure 5.1.1. The series may either converge or diverge when \( |x - x_0| = \rho \).

**FIGURE 5.1.1** The interval of convergence of a power series.

Determine the radius of convergence of the power series

\[
\sum_{n=1}^{\infty} \frac{(x + 1)^n}{n2^n}.
\]

We apply the ratio test:

\[
\lim_{n \to \infty} \left| \frac{(x + 1)^{n+1}}{(n + 1)2^{n+1}} \frac{n2^n}{(x + 1)^n} \right| = \frac{|x + 1|}{2} \lim_{n \to \infty} \frac{n}{n + 1} = \frac{|x + 1|}{2}.
\]

Thus the series converges absolutely for \( |x + 1| < 2 \), or \(-3 < x < 1\), and diverges for \( |x + 1| > 2 \). The radius of convergence of the power series is \( \rho = 2 \). Finally, we check the endpoints of the interval of convergence. At \( x = 1 \) the series becomes the harmonic series

\[
\sum_{n=1}^{\infty} \frac{1}{n},
\]

which diverges. At \( x = -3 \) we have

\[
\sum_{n=1}^{\infty} \frac{(-3 + 1)^n}{n2^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n},
\]

which converges, but does not converge absolutely. The series is said to converge conditionally at \( x = -3 \). To summarize, the given power series converges for \(-3 \leq x < 1\), and diverges otherwise. It converges absolutely for \(-3 < x < 1\), and has a radius of convergence 2.

If \( \sum_{n=0}^{\infty} a_n(x - x_0)^n \) and \( \sum_{n=0}^{\infty} b_n(x - x_0)^n \) converge to \( f(x) \) and \( g(x) \), respectively, for \( |x - x_0| < \rho \), \( \rho > 0 \), then the following are true for \( |x - x_0| < \rho \).

6. The series can be added or subtracted termwise and

\[
f(x) \pm g(x) = \sum_{n=0}^{\infty} (a_n \pm b_n)(x - x_0)^n.
\]