Supplementary Problems

PROBABILITY VECTORS AND STOCHASTIC MATRICES

20.31. Which vectors are probability vectors?
   (i) \((\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2})\)   (ii) \((\frac{1}{2}, 0, \frac{1}{2}, \frac{1}{2})\)   (iii) \((\frac{1}{2}, \frac{1}{2}, 0, \frac{1}{2})\).

20.32. Find a scalar multiple of each vector which is a probability vector:
   (i) \((3, 0, 2, 5, 3)\)   (ii) \((2, \frac{1}{2}, 0, \frac{1}{2}, 0, 1)\)   (iii) \((\frac{1}{4}, 2, \frac{1}{2}, 0, \frac{1}{4})\).

20.33. Which matrices are stochastic?
   (i) \(\begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}\)   (ii) \(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\)   (iii) \(\begin{pmatrix} 0 & 1 \\ \frac{1}{2} \end{pmatrix}\)   (iv) \(\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}\)   (v) \(\begin{pmatrix} 0 & 1 \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}\).

REGULAR STOCHASTIC MATRICES AND FIXED PROBABILITY VECTORS

20.34. Find the unique fixed probability vector of each matrix:
   (i) \(\begin{pmatrix} \frac{3}{8} & \frac{1}{8} \\ \frac{5}{8} & \frac{3}{8} \end{pmatrix}\)   (ii) \(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\)   (iii) \(\begin{pmatrix} .2 & .8 \\ .5 & .5 \end{pmatrix}\)   (iv) \(\begin{pmatrix} .7 & .3 \\ .6 & .4 \end{pmatrix}\).

20.35. (i) Find the unique fixed probability vector \(t\) of \(P = \begin{pmatrix} 0 & \frac{3}{4} \\ \frac{1}{4} & 0 \end{pmatrix} \).
   (ii) What matrix does \(P^n\) approach?   (iii) What vector does \((\frac{1}{4}, \frac{1}{2}, \frac{1}{4})P^n\) approach?

20.36. Find the unique fixed probability vector \(t\) of each matrix:
   (i) \(A = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}\)   (ii) \(B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\).

20.37. (i) Find the unique fixed probability vector \(t\) of \(P = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix} \).
   (ii) What matrix does \(P^n\) approach?
   (iii) What vector does \((\frac{1}{2}, 0, \frac{1}{2})P^n\) approach?
   (iv) What vector does \((\frac{1}{2}, 0, \frac{1}{2})P^n\) approach?

20.38. (i) Given that \(t = (\frac{1}{2}, 0, \frac{1}{2})\) is a fixed point of a stochastic matrix \(P\), is \(P\) regular?
   (ii) Given that \(t = (\frac{1}{4}, \frac{1}{2}, \frac{1}{4})\) is a fixed point of a stochastic matrix \(P\), is \(P\) regular?

20.39. Which of the stochastic matrices are regular?
   (i) \(\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & 1 \end{pmatrix}\)   (ii) \(\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}\)   (iii) \(\begin{pmatrix} 0 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}\)

20.40. Show that \((af + ce + de, af + bf + ae, ad + bd + bc)\) is a fixed point of the matrix
   \[
P = \begin{pmatrix} 1-a-b & a & b \\ c & 1-c-d & d \\ e & f & 1-e-f \end{pmatrix}\]