242 BINOMIAL COEFFICIENTS

Research problems

95 Let \( q(n) \) be the smallest odd prime factor of the middle binomial coefficient \( \binom{2n}{n} \). According to exercise 36, the odd primes \( p \) that do not divide \( \binom{2n}{n} \) are those for which all digits in \( n \)'s radix \( p \) representation are \((p - 1)/2\) or less. Computer experiments have shown that \( q(n) \leq 11 \) for all \( n < 10^{10000} \), except that \( q(3160) = 13 \).
   a. Is \( q(n) \leq 11 \) for all \( n > 3160 \)?
   b. Is \( q(n) = 11 \) for infinitely many \( n \)?

A reward of \( \$\) is offered for a solution to either (a) or (b).

96 Is \( \binom{2n}{n} \) divisible by the square of a prime, for all \( n > 4 \)?

97 For what values of \( n \) is \( \binom{2n}{n} \equiv (-1)^n \pmod{2n + 1} \)?