Stirling numbers of the second kind show up more often than those of the other variety, so let’s consider last things first. The symbol \( \{n\} \) stands for the number of ways to partition a set of \( n \) things into \( k \) nonempty subsets.

For example, there are seven ways to split a four-element set into two parts:

\[
\{1, 2, 3, 4\}, \quad \{1, 2, 4\} \cup \{3\}, \quad \{1, 3, 4\} \cup \{2\}, \quad \{2, 3, 4\} \cup \{1\}, \\
\{1, 2\} \cup \{3, 4\}, \quad \{1, 3\} \cup \{2, 4\}, \quad \{1, 4\} \cup \{2, 3\};
\]

thus \( \{4\} = 7 \). Notice that curly braces are used to denote sets as well as the numbers \( \{n\} \). This notational kinship helps us remember the meaning of \( \{n\} \), which can be read “\( n \) subset \( k \).”

Let’s look at small \( k \). There’s just one way to put \( n \) elements into a single nonempty set; hence \( \{1\} = 1 \), for all \( n > 0 \). On the other hand \( \{0\} = 0 \), because a 0-element set is empty.

The case \( k = 0 \) is a bit tricky. Things work out best if we agree that there’s just one way to partition an empty set into zero nonempty parts; hence \( \{0\} = 1 \). But a nonempty set needs at least one part, so \( \{n\} = 0 \) for \( n > 0 \).

What happens when \( k = 2 \)? Certainly \( \{0\} = 0 \). If a set of \( n > 0 \) objects is divided into two nonempty parts, one of those parts contains the last object and some subset of the first \( n - 1 \) objects. There are \( 2^{n-1} \) ways to choose the latter subset, since each of the first \( n - 1 \) objects is either in it or out of it; but we mustn’t put all of those objects in it, because we want to end up with two nonempty parts. Therefore we subtract 1:

\[
\{n\} \atop {2} = 2^{n-1} - 1, \quad \text{integer } n > 0.
\]

(This tallies with our enumeration of \( \{4\} = 7 = 2^3 \) 1 ways above.)