focusing the particles on the peaks of $g$. Note however that, unlike the other resampling methods described above, weighted resampling does not assign equal weights $(1/N)$ to all the particles. In the rest of the paper, we will need this “equal weight” feature, so whenever weighted resampling will be used, it will be implicitly followed by one of the other above resampling methods.

In the next section, we will propose a new resampling method that exploits the structure within articulated objects to improve the efficiency of particle filtering.

4 DBN-BASED COMBINATORIAL RESAMPLING

Our resampling scheme is suitable for particle filters as described in Alg. 1. More precisely, we will show in Subsection 4.1 that, in articulated object tracking, the set $\{1, \ldots, P\}$ of parts of the objects to track can be partitioned into some sets $\{P_1, \ldots, P_K\}$ such that those parts in each $P_j$ are all independent conditionally to $\cup_{h<j} P_h$. For instance, in Fig. 2, $P = 6$ and $K = 3$, $P_1 = \{1\}$ corresponds to the torso, $P_2 = \{2, 4, 6\}$ to the head and both arms, and $P_3 = \{3, 5\}$ to the forearms. In addition, given the position of the torso ($P_1$), those of the head and the arms ($P_2$) are independent. In Subsection 4.2, these independences will be exploited to justify that permutations of some particles’ parts do not alter the estimation of $p(x_{1:t}|y_{1:t})$. Then, our resampling scheme, which will be described in Subsection 4.3, will exploit these permutations to construct implicitly some new exponential-size sample from which it will resample new high-quality samples.

4.1 IDENTIFYING SETS $P_1, \ldots, P_K$

To be sound, i.e., to not alter the estimation of $p(x_{1:t}|y_{1:t})$, Combinatorial Resampling exploits conditional independences among the different parts of the object. The partition into sets $P_1, \ldots, P_K$ precisely accounts for these independences and thus naturally results from a d-separation analysis, the independence property at the core of DBNs:

**Definition 1 (d-separation [Pearl, 1988])** Two nodes $x_i^t$ and $x_j^t$ of a DBN are dependent conditionally to a set of nodes $Z$ if and only if there exists a chain, i.e., an undirected path, $\{c_1 = x_i^t, \ldots, c_n = x_j^t\}$ linking $x_i^t$ and $x_j^t$ in the DBN such that the following two conditions hold:

1. for every node $c_k$ such that the arcs are $c_{k-1} \rightarrow c_k \leftarrow c_{k+1}$, either $c_k$ or one of its descendants is in $Z$;
2. none of the other nodes $c_h$ belongs to $Z$.

Such a chain is called active (else it is blocked). If there exists an active chain linking two nodes, these nodes are dependent and are called d-connected, otherwise they are independent conditionally to $Z$ and are called d-separated.

In Fig. 2, conditionally to the position of the torso up to time $t$, both arms are thus independent.

In the rest of the paper, we will assume that, **within each time slice**, the DBN structure is a directed tree, i.e., there do not exist nodes $x_i^t, x_j^t, x_k^t$ such that $x_i^t \rightarrow x_j^t \leftarrow x_k^t$. We will also assume that arcs across time slices link similar nodes, i.e., there exist no arc $x_i^{t-1} \rightarrow x_j^t$ with $j \neq i$. Finally, we will assume that nodes $y_i^t$ have only one parent $x_i^t$ and no children. For articulated object tracking, these requirements are rather mild and Fig. 2 satisfies all of them.

Now, we can construct sets $P_1, \ldots, P_K$: for any node, say $X_t$, in time slice $t$ of the DBN, let $Pa(X_t)$ and $Pa_i(X_t)$ denote the set of parents of $X_t$ in the DBN in all time slices and in time slice $t$ only respectively. For instance, in Fig. 2, $Pa_i(x_3^t) = \{x_1^t, x_2^t\}$ and $Pa_i(x_4^t) = \{x_1^t\}$. Let $\{P_1, \ldots, P_K\}$ be a partition of $\{1, \ldots, P\}$ defined by:

- $P_1 = \{k \in \{1, \ldots, P\} : Pa_i(x_k^t) = \emptyset\}$;
- for any $j > 1$, $P_j = \{k \in \{1, \ldots, P\} \setminus \bigcup_{h=1}^{j-1} P_h : Pa_i(x_k^t) \subseteq \bigcup_{h=1}^{j-1} \bigcup_{r \in P_h} \{x_i^t\}\}$.

On Fig. 2, this results in $P_1 = \{1\}$, $P_2 = \{2, 4, 6\}$ and $P_3 = \{3, 5\}$. It turns out that the way we constructed the $P_j$’s, all the $x_i^t \in P_j$ can be processed independently by PF because they are independent conditionally to $Pa_i(x_k^t)$, and this is precisely this independence property which is needed to enable a sound object part swapping within Combinatorial Resampling:

**Proposition 1** The particle set resulting from Algorithm 1, with $P_j$ defined as in the preceding paragraph, $Q_j = \sum_{h=1}^{j} P_h$ and $R_j = \sum_{h=j+1}^{P} P_h$, represents $p(x_{1:t}|y_{1:t})$.

**Proof:** By induction on $j$. Assume that, before processing parts $P_j$, particles estimate $p(x_{1:t-1}^{Q_{j-1}}, x_{t-1}^{R_{j-1}}|y_{1:t-1}, y_{t}^{Q_{j-1}})$. This is clearly the case for $P_1$ since $P_1$ are the first parts processed. Remember that $P_j, Q_j, R_j$ are the set of parts processed at the $j$th step and still to process respectively. We will now examine sequentially the distributions estimated by the particle set after applying in parallel PF’s prediction step over the parts in $P_j$, then after applying PF’s correction step and, finally, after resampling.

1. Let us show that after the parallel propagations of the parts in $P_j$ (prediction step), the particle set represents density $p(x_{1:t}^{Q_{j}}; x_{t-1}^{R_{j-1}}|y_{1:t-1}, y_{t}^{Q_{j-1}})$. For instance, on Fig. 2, this means that, after propagating the parts in $P_2$, the particle set estimates $p(x_{1:t}^{2, 4, 6}, x_{t-1}^{3, 5}|y_{1:t-1}, y_{t}^{2, 4, 6})$, i.e., only the positions of the forearms still refer to time $t-1$ and the only observation taken into account at time $t$ is the position of the torso (not yet those of the head and arms). All these