the other one will contain a configuration that violates $b/2$ constraints. Therefore we conclude that there is an energy barrier of “height” $b/2$ between the two solutions.

For large values of $b$, one cannot find a temperature $T$ that provides both high enough $P_T(0)$ (i.e. small number of flips per solution) and $P_T(b/2)$ (i.e. probability of “jumping” over the barrier) to obtain a practical sampler. For instance, for $b = 80$, to get $P_T(40) > 10^{-3}$ (i.e. climbing the barrier on average once in about a billion samples) SA needs a temperature $T > 0.75$, but $P_{0.75}(0) = 7.4 \times 10^{-9}$ (which means that on average we need more than $10^8$ samples to get a single solution). Similar results are obtained when using Gibbs sampling. This is because a Gibbs sampler has the same Boltzmann steady state probability distribution and it also proceeds by flipping a single variable at a time, so it cannot “jump” over the barrier with a single move.

Since instances in $XORBarrier(b)$ belong to 2-SAT and have 2 solutions, from Theorem 2 Algorithm 1 provides uniform samples in polynomial time.

A common strategy to partially overcome the ergodicity problems of Gibbs sampling and Simulated Annealing is the use of multiple parallel chains [8] or restarts [9]. These approaches can be quite effective and, for instance, are capable of producing uniform samples for the $XORBarrier(b)$ class of instances presented above. Intuitively, this is because on average 50% of the random initial assignments will be on one side of the barrier (i.e., the half-hypercube with $x_1 = 0$) and the other 50% on the other side (i.e., $x_1 = 1$).

### 6.4 Embedded energy barriers

In Table 1 (top rows), we evaluate the performance of the methods considered on three types of instances: a logistic one generated by SATPlan [19] (logistic, with 110 variables, 461 clauses, and 512 solutions), a graph Coloring problem from SatLib [20](coloring, with 90 variables, 300 clauses, and 900 solutions), and a random 3-SAT formula (random, with 75 variables, 315 clauses, and 48 solutions). We collect respectively $P = 50000$, $P = 200000$, and $P = 5000$ samples for each instance. We also artificially introduce an energy barrier in each of these instances by choosing a variable $z$ such that there are roughly half solutions with $z = 1$ and half solutions with $z = 0$ in the original formula. We then introduce a new set clauses defined by $XORBarrier(40)$ where we substitute $z$ to $x_1$ and $y_1, \ldots, y_{10}$ are fresh variables. Note that this does not change the number of solutions.

We experimented with several values of $k$ and $T$ and we provide a summary of the best results obtained in Table 1, both for the original formulas and the ones with an embedded energy barrier. By carefully choosing the tempera-