each of the following sentences is true:
\[ \alpha_2 = \text{"There is no pit in [2,2]."} \]
\[ \alpha_3 = \text{"There is a wumpus in [1,3]."} \]

Hence show that \( KB \) if \( \alpha_2 \) and \( KB \) if \( \alpha_3 \).

7.2 (Adapted from Barwise and Etchemendy (1993).) Given the following, can you prove that the unicorn is mythical? How about magical? Horned?

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned.

The unicorn is magical if it is horned.

7.3 Consider the problem of deciding whether a propositional logic sentence is true in a given model.

a. Write a recursive algorithm \( \text{PL-TRUE?}(s, m) \) that returns \texttt{true} if and only if the sentence \( s \) is true in the model \( m \) (where \( \mathit{sit} \) assigns a truth value for every symbol in \( s \)). The algorithm should run in time linear in the size of the sentence. (Alternatively, use a version of this function from the online code repository.)

b. Give three examples of sentences that can be determined to be true or false in a partial model that does not specify a truth value for some of the symbols.

c. Show that the truth value (if any) of a sentence in a partial model cannot be determined efficiently in general.

d. Modify your \( \text{PL-TRUE?} \) algorithm so that it can sometimes judge truth from partial models, while retaining its recursive structure and linear run time. Give three examples of sentences whose truth in a partial model is not detected by your algorithm.

e. Investigate whether the modified algorithm makes \( \text{TT-ENTAILS?} \) more efficient.

7.4 Which of the following are correct?

a. \( \text{False} \models \text{True} \).

b. \( \text{True} \not\models \text{False} \).

c. \( (A \land B) \not\models (A \equiv B) \).

d. \( A \).

e. \( A \lor B \implies \neg A \lor B \).

f. \( (A \land B) \lor (C \land (A \land C) \lor (B \lor C)) \).

g. \( (C \lor (A \land B)) \models ((A \lor C) \land (B \lor C)) \).

h. \( (A \lor B) \land (\neg C \lor \neg D \lor E) \models (A \lor B) \).

i. \( (A \lor B) \lor (C \lor D \lor E) \models (A \lor B) \land (C \lor D \lor E) \).

j. \( (A \lor B) \land (A \implies B) \) is satisfiable.

k. \( (A \equiv B) \land (\neg A \lor B) \) is satisfiable.

l. \( (A \lor B) \land (A \lor B) \) has the same number of models as \( (A \equiv B) \) for any fixed set of proposition symbols that includes \( A, B, C \).
Exercises

7.5 Prove each of the following assertions:
- **a.** \( a \) is valid if and only if \( \text{True} \).
- **b.** For any \( a, \text{False} \).
- **c.** \( a \equiv \beta \) if and only if the sentence \( (a \equiv \beta) \) is valid.
- **d.** \( a \not\equiv \beta \) if and only if the sentence \( (a \not\equiv \beta) \) is valid.
- **e.** \( a \) if and only if the sentence \( (a \not\beta) \) is unsatisfiable.

7.6 Prove, or find a counterexample to, each of the following assertions:
- **a.** If \( a \) or \( \beta \) (or both) then \( \gamma \).
- **b.** If \( a \) if \( (\beta \land \gamma) \) then \( a \) and \( \beta \) and \( \gamma \).
- **c.** If \( a \) then \( \beta \) or \( \gamma \) (or both).

7.7 Consider a vocabulary with only four propositions, \( A, B, C, \) and \( D \). How many models are there for the following sentences?
- **a.** \( B \lor C \).
- **b.** \( A \lor \neg B \lor \neg C \lor \neg D \).
- **c.** \( (A = B) \land A \land \neg B \lor C \lor D \).

7.8 We have defined four binary logical connectives.
- **a.** Are there any others that might be useful?
- **b.** How many binary connectives can there be?
- **c.** Why are some of them not very useful?

7.9 Using a method of your choice, verify each of the equivalences in Figure 7.11 (page 249).

7.19 Decide whether each of the following sentences is valid, unsatisfiable, or neither. Verify your decisions using truth tables or the equivalence rules of Figure 7.11 (page 249).
- **a.** \( \text{Smoke} = \text{Smoke} \)
- **b.** \( \text{Smoke} = \text{Fire} \)
- **c.** \( (\text{Smoke} \land \text{Fire}) = (\neg \text{Smoke} \lor \text{Fire}) \)
- **d.** \( \text{Smoke} \lor \text{Fire} \lor \neg \text{Fire} \)
- **e.** \( ((\text{Smoke} \land \text{Heat}) = \text{Fire}) \lor ((\text{Smoke} = \text{Fire}) \lor \text{Heat} = \text{Fire}) \)
- **f.** \( (\text{Smoke} \lor \text{Fire}) \lor (\text{Smoke} \land \text{Heat} \lor \text{Fire}) \)
- **g.** \( \text{Big} \lor \text{Dumb} \lor (\text{Big} \land \text{Dumb}) \)

7.11 Any propositional logic sentence is logically equivalent to the assertion that each possible world in which it would be false is not the case. From this observation, prove that any sentence can be written in \textit{CNF}.

7.12 Use resolution to prove the sentence \( \neg B \) from the clauses in Exercise 7.20.

7.13 This exercise looks into the relationship between clauses and implication sentences.