The continuant polynomial $K_n(x_1, x_2, \ldots, x_n)$ has $n$ parameters, and it is defined by the following recurrence:

\begin{align*}
K_0() &= 1; \\
K_1(x_1) &= x_1; \\
K_n(x_1, \ldots, x_n) &= K_{n-1}(x_1, \ldots, x_{n-1})x_n + K_{n-2}(x_1, \ldots, x_{n-2}).
\end{align*} \quad (6.127)

For example, the next three cases after $K_1(x_1)$ are

\begin{align*}
K_2(x_1, x_2) &= x_1x_2 + 1; \\
K_3(x_1, x_2, x_3) &= x_1x_2x_3 + x_1 + x_3; \\
K_4(x_1, x_2, x_3, x_4) &= x_1x_2x_3x_4 + x_1x_2 + x_1x_4 + x_3x_4 + 1.
\end{align*}

It's easy to see, inductively, that the number of terms is a Fibonacci number:

$$K_n(1, 1, \ldots, 1) = F_{n+1}. \quad (6.128)$$

When the number of parameters is implied by the context, we can write simply 'K' instead of 'K,' just as we can omit the number of parameters when we use the hypergeometric functions $F$ of Chapter 5. For example, $K(x_1, x_2) = K_2(x_1, x_2) = x_1x_2 + 1$. The subscript $n$ is of course necessary in formulas like (6.128).

Euler observed that $K(x_1, x_2, \ldots, x_n)$ can be obtained by starting with the product $x_1x_2\ldots x_n$ and then striking out adjacent pairs $x_kx_{k+1}$ in all possible ways. We can represent Euler’s rule graphically by constructing all “Morse code” sequences of dots and dashes having length $n$, where each dot contributes 1 to the length and each dash contributes 2; here are the Morse code sequences of length 4:

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..-  .-..  -..  --
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These dot-dash patterns correspond to the terms of $K(x_1, x_2, x_3, x_4)$; a dot signifies a variable that’s included and a dash signifies a pair of variables that’s excluded. For example, ..- corresponds to $x_1x_4$.

A Morse code sequence of length $n$ that has $k$ dashes has $n-2k$ dots and $n-k$ symbols altogether. These dots and dashes can be arranged in \(\binom{n-k}{k}\) ways; therefore if we replace each dot by $z$ and each dash by 1 we get

$$K_n(z, z, \ldots, z) = \sum_{k=0}^{n} \binom{n-k}{k}z^{n-2k} \quad (6.129)$$