The continuant polynomial \( K_n(x_1, x_2, \ldots, x_n) \) has \( n \) parameters, and it is defined by the following recurrence:

\[
K_0() = 1; \quad K_1(x_1) = x_1; \quad K_n(x_1, \ldots, x_n) = K_{n-1}(x_1, \ldots, x_{n-1}) x_n + K_{n-2}(x_1, \ldots, x_{n-2}). \quad (6.127)
\]

For example, the next three cases after \( K_1(x_1) \) are

\[
K_2(x_1, x_2) = x_1 x_2 + 1; \quad K_3(x_1, x_2, x_3) = x_1 x_2 x_3 + x_1 + x_3; \quad K_4(x_1, x_2, x_3, x_4) = x_1 x_2 x_3 x_4 + x_1 x_2 + x_1 x_4 + x_3 x_4 + 1
\]

It’s easy to see, inductively, that the number of terms is a Fibonacci number:

\[
K_n(1, 1, \ldots, 1) = F_{n+1}. \quad (6.128)
\]

When the number of parameters is implied by the context, we can write simply ‘\( K \)’ instead of ‘\( K_n \)’, just as we can omit the number of parameters when we use the hypergeometric functions \( F \) of Chapter 5. For example, \( K(x_1, x_2) = K_2(x_1, x_2) = x_1 x_2 + 1 \). The subscript \( n \) is of course necessary in formulas like (6.128).

Euler observed that \( K(x_1, x_2, \ldots, x_n) \) can be obtained by starting with the product \( x_1 x_2 \ldots x_n \) and then striking out adjacent pairs \( x_k x_{k+1} \) in all possible ways. We can represent Euler’s rule graphically by constructing all “Morse code” sequences of dots and dashes having length \( n \), where each dot contributes 1 to the length and each dash contributes 2; here are the Morse code sequences of length 4:

\[
\ldots \ldots \ldots \ldots \; \ldots 
\]

These dot-dash patterns correspond to the terms of \( K(x_1, x_2, x_3, x_4) \); a dot signifies a variable that’s included and a dash signifies a pair of variables that’s excluded. For example, \( \ldots \ldots \) corresponds to \( x_1 x_4 \).

A Morse code sequence of length \( n \) that has \( k \) dashes has \( n-2k \) dots and \( n-k \) symbols altogether. These dots and dashes can be arranged in \( \binom{n-k}{k} \) ways; therefore if we replace each dot by \( z \) and each dash by 1 we get

\[
K_n(z, z, \ldots, z) = \sum_{k=0}^{n} \binom{n-k}{k} z^{n-2k} \quad (6.129)
\]