Thus we can convert at sight between continued fractions and the corresponding nodes in the Stern-Brocot tree. For example,

\[
f(\text{LRRL}) = 0 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1}}}}.
\]

We observed in Chapter 4 that irrational numbers define infinite paths in the Stern-Brocot tree, and that they can be represented as an infinite string of L’s and R’s. If the infinite string for \(a\) is \(R \alpha_1 L \alpha_2 R \alpha_3 L \alpha_4 \ldots\), there is a corresponding infinite continued fraction

\[
a = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{a_4 + \cfrac{1}{\ddots}}}}}. \tag{6.141}
\]

This infinite continued fraction can also be obtained directly: Let \(a_0 = a\) and for \(k \geq 0\) let

\[
a_k = \lfloor a_k \rfloor; \quad a_k = a + \cfrac{1}{a_{k+1}}. \tag{6.142}
\]

The \(a_k\)’s are called the “partial quotients” of \(a\). If \(a\) is rational, say \(m/n\), this process runs through the quotients found by Euclid’s algorithm and then stops (with \(a_{k+1} = \infty\)).

Is Euler’s constant \(\gamma\) rational or irrational? Nobody knows. We can get partial information about this famous unsolved problem by looking for \(\gamma\) in the Stern-Brocot tree; if it’s rational we will find it, and if it’s irrational we will find all the closest rational approximations to it. The continued fraction for \(\gamma\) begins with the following partial quotients:

\[
\begin{array}{cccccccc}
  k & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
  a_k & 0 & 1 & 1 & 2 & 1 & 2 & 1 & 4 & 3
\end{array}
\]

Therefore its Stern-Brocot representation begins \(LRLLRLLRLRLRLRRL\ldots\); no pattern is evident. Calculations by Richard Brent [33] have shown that, if \(\gamma\) is rational, its denominator must be more than 10,000 decimal digits long.