can have any degree of belief, ranging from 0 (total disbelief) to 1 (total belief). For example, a probabilistic wumpus-world agent might believe that the wumpus is in [1,3] with probability 0.75. The ontological and epistemological commitments of five different logics are summarized in Figure 8.1.

<table>
<thead>
<tr>
<th>Language</th>
<th>Ontological Commitment (What exists in the world)</th>
<th>Epistemological Commitment (What an agent believes about facts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propositional logic</td>
<td>facts</td>
<td>true/false/unknown</td>
</tr>
<tr>
<td>First-order logic</td>
<td>facts, objects, relations</td>
<td>true/false/unknown</td>
</tr>
<tr>
<td>Temporal logic</td>
<td>facts, objects, relations, times</td>
<td>true/false/unknown</td>
</tr>
<tr>
<td>Probability theory</td>
<td>facts</td>
<td>degree of belief E ∈ [0, 1]</td>
</tr>
<tr>
<td>Fuzzy logic</td>
<td>facts with degree of truth E ∈ [0, 1]</td>
<td>known interval value</td>
</tr>
</tbody>
</table>

Figure 8.1 Formal languages and their ontological and epistemological commitments.

In the next section, we will launch into the details of first-order logic. Just as a student of physics requires some familiarity with mathematics, a student of AI must develop a talent for working with logical notation. On the other hand, it is also important not to get too concerned with the specifics of logical notation—after all, there are dozens of different versions. The main things to keep hold of are how the language facilitates concise representations and how its semantics leads to sound reasoning procedures.

8.2 SYNTAX AND SEMANTICS OF FIRST-ORDER LOGIC

We begin this section by specifying more precisely the way in which the possible worlds of first-order logic reflect the ontological commitment to objects and relations. Then we introduce the various elements of the language, explaining their semantics as we go along.

8.2.1 Models for first order logic

Recall from Chapter 7 that the models of a logical language are the formal structures that constitute the possible worlds under consideration. Each model links the vocabulary of the logical sentences to elements of the possible world, so that the truth of any sentence can be determined. Thus, models for propositional logic link proposition symbols to predefined truth values. Models for first-order logic are much more interesting. First, they have objects in them! The domain of a model is the set of objects or domain elements it contains. The domain is required to be non-empty—every possible world must contain at least one object. (See Exercise 8.7 for a discussion of empty worlds.) Mathematically speaking, it doesn’t matter what these objects are—all that matters is how many there are in each particular model—but for pedagogical purposes we will use a concrete example. Figure 8.2 shows a model with five...
objects: Richard the Lionheart, King of England from 1189 to 1199; his younger brother, the evil King John, who ruled from 1199 to 1215; the left legs of Richard and John; and a crown.

The objects in the model may be related in various ways. In the figure, Richard and John are brothers. Formally speaking, a relation is just the set of tuples of objects that are related. (A tuple is a collection of objects arranged in a fixed order and is written with angle brackets surrounding the objects.) Thus, the brotherhood relation in this model is the set

\[
\text{Richard the Lionheart, King John, (King John, Richard the Lionheart)} \quad (8.1)
\]

(Here we have named the objects in English, but you may, if you wish, mentally substitute the pictures for the names.) The crown is on King John’s head, so the “on head” relation contains just one tuple. (the crown, King John). The ”brother” and ”on head” relations are binary relations—that is, they relate pairs of objects. The model also contains unary relations, or properties: the ”person” property is true of both Richard and John; the ”king” property is true only of John (presumably because Richard is dead at this point); and the ”crown” property is true only of the crown.

Certain kinds of relationships are best considered as functions, in that a given object must be related to exactly one object in this way. For example, each person has one left leg, so the model has a unary ”left leg” function that includes the following mappings:

\[
\begin{align*}
\text{Richard the Lionheart} & \rightarrow \text{Richard’s left leg} \\
\text{King John} & \rightarrow \text{John’s left leg}.
\end{align*}
\]

(8.2)

Strictly speaking, models in first-order logic require total functions, that is, there must be a value for every input tuple. Thus, the crown must have a left leg and so must each of the left legs. There is a technical solution to this awkward problem involving an additional ”invisible”