16 What is the general solution of the double recurrence

\[ A_{n,0} = a_n \text{ for } n \geq 0; \quad A_{0,k} = 0, \text{ if } k > 0; \]
\[ A_{n,k} = kA_{n-1,k} + A_{n-1,k-1}, \]
for integers \( k, n \) when \( k \) and \( n \) range over the set of all integers?

17 Solve the following recurrences, assuming that \( \binom{n}{k} \) is zero when \( n < 0 \) or \( k < 0 \):

a. \[ \binom{n}{k} = \binom{n-1}{k} + \binom{n}{k-1} + \binom{n}{k} = 0, \quad \text{for } n, k \geq 0. \]

b. \[ \binom{n}{k} = (n-k)\binom{n-1}{k} + \binom{n-1}{k-1} + \binom{n}{k} = 0, \quad \text{for } n, k \geq 0. \]

c. \[ \binom{n}{k} = k\binom{n-1}{k} + k\binom{n}{k-1} + \binom{n}{k} = 0, \quad \text{for } n, k \geq 0. \]

18 Prove that the Stirling polynomials satisfy

\[ (x+1)\sigma_n(x+1) = (x-n)\sigma_n(x) + x\sigma_{n-1}(x) \]

19 Prove that the generalized Stirling numbers satisfy

\[ \sum_{k=0}^{n} \binom{x+k}{x} \binom{x}{x-n+k} (-1)^k \binom{x+k}{n+1} = 0, \text{ integer } n > 0. \]

\[ \sum_{k=0}^{n} \binom{x+k}{x} \binom{x}{x-n+k} (-1)^k \binom{x+k}{n+1} = 0, \text{ integer } n > 0. \]

20 Find a closed form for \( \sum_{k=1}^{n} H_k^{(2)} \).

21 Show that if \( H_n = \frac{a_n}{b_n} \), where \( a_n \) and \( b_n \) are integers, the denominator \( b_n \) is a multiple of \( 2^{|\log n|} \). Hint: Consider the number \( 2^{|\log n|} H_n \rightarrow \frac{1}{2} \).

22 Prove that the infinite sum

\[ \sum_{k=1}^{\infty} \left( \frac{1}{k} - \frac{1}{k+z} \right) \]

converges for all complex numbers \( z \), except when \( z \) is a negative integer; and show that it equals \( H_z \) when \( z \) is a nonnegative integer. (Therefore we can use this formula to define harmonic numbers \( H_z \) when \( z \) is complex.)

23 Equation (6.81) gives the coefficients of \( z/(e^z - 1) \), when expanded in powers of \( z \). What are the coefficients of \( z/(e^z + 1) \)? Hint: Consider the identity \( (e^z + 1)/(e^z - 1) = e^{2z} - 1 \).