Let \( f(t) = t^{-p}, \ t \geq 1, \) where \( p \) is a real constant and \( p \neq 1; \) the case \( p = 1 \) was considered in Example 2. Then
\[
\int_1^\infty t^{-p} \, dt = \lim_{A \to \infty} \int_1^A t^{-p} \, dt = \lim_{A \to \infty} \frac{1}{1-p} (A^{1-p} - 1).
\]
As \( A \to \infty, \ A^{1-p} \to 0 \) if \( p > 1, \) but \( A^{1-p} \to \infty \) if \( p < 1. \) Hence \( \int_1^\infty t^{-p} \, dt \) converges for \( p > 1, \) but (incorporating the result of Example 2) diverges for \( p \leq 1. \) These results are analogous to those for the infinite series \( \sum_{n=1}^{\infty} n^{-p}. \)

Before discussing the possible existence of \( \int_a^\infty f(t) \, dt, \) it is helpful to define certain terms. A function \( f \) is said to be \textbf{piecewise continuous} on an interval \( \alpha \leq t \leq \beta \) if the interval can be partitioned by a finite number of points \( \alpha = t_0 < t_1 < \cdots < t_n = \beta \) so that
\begin{enumerate}
  \item \( f \) is continuous on each open subinterval \( t_{i-1} < t < t_i. \)
  \item \( f \) approaches a finite limit as the endpoints of each subinterval are approached from within the subinterval.
\end{enumerate}

In other words, \( f \) is piecewise continuous on \( \alpha \leq t \leq \beta \) if it is continuous there except for a finite number of jump discontinuities. If \( f \) is piecewise continuous on \( \alpha \leq t \leq \beta \) for every \( \beta > \alpha, \) then \( f \) is said to be piecewise continuous on \( t \geq \alpha. \) An example of a piecewise continuous function is shown in Figure 6.1.1.

If \( f \) is piecewise continuous on the interval \( \alpha \leq t \leq A, \) then it can be shown that \( \int_\alpha^A f(t) \, dt \) exists. Hence, if \( f \) is piecewise continuous for \( t \geq \alpha, \) then \( \int_\alpha^A f(t) \, dt \) exists for each \( A > \alpha. \) However, piecewise continuity is not enough to ensure convergence of the improper integral \( \int_\alpha^\infty f(t) \, dt, \) as the preceding examples show.

If \( f \) cannot be integrated easily in terms of elementary functions, the definition of convergence of \( \int_\alpha^\infty f(t) \, dt \) may be difficult to apply. Frequently, the most convenient way to test the convergence or divergence of an improper integral is by the following comparison theorem, which is analogous to a similar theorem for infinite series.

\[\text{FIGURE 6.1.1} \quad \text{A piecewise continuous function.}\]