33 Table 250 gives the values of $\binom{n}{2}$ and $\{n\}$. What are closed forms (not involving Stirling numbers) for the next cases, $\binom{n}{3}$ and $\{n\}$?

34 What are $\{-\frac{1}{k}\}$ and $\{-\frac{2}{k}\}$, if the basic recursion relation (6.35) is assumed to hold for all integers $k$ and $n$, and if $\binom{n}{k} = 0$ for all $k < 0$?

35 Prove that, for every $\epsilon > 0$, there exists an integer $n > 1$ (depending on $\epsilon$) such that $H_n \mod 1 < \epsilon$.

36 Is it possible to stack $n$ bricks in such a way that the topmost brick is not above any point of the bottommost brick, yet a person who weighs the same as 100 bricks can balance on the middle of the top brick without toppling the pile?

37 Express $\sum_{k=1}^{m} (k \mod m)/k(k+1)$ in terms of harmonic numbers, assuming that $m$ and $n$ are positive integers. What is the limiting value as $n \to \infty$?

38 Find the indefinite sum $\sum \binom{n}{k} (-1)^k H_k \delta k$.

39 Express $\sum_{k=1}^{n} H_k^2$ in terms of $n$ and $H_n$.

40 Prove that 1979 divides the numerator of $\sum_{k=1}^{1979} (-1)^{k-1}/k$, and give a similar result for 1987. Hint: Use Gauss's trick to obtain a sum of fractions whose numerators are 1979. See also exercise 4.

41 Evaluate the sum

$$\sum_k \binom{\lceil (n+k)/2 \rceil}{k}$$

in closed form, when $n$ is an integer (possibly negative).

42 If $S$ is a set of integers, let $S + 1$ be the “shifted” set $\{x + 1 \mid x \in S\}$.

How many subsets of $\{1, 2, \ldots, n\}$ have the property that $S \cup (S + 1) = \{1, 2, \ldots, n+1\}$?

43 Prove that the infinite sum

$$.1$$
$$+.01$$
$$+.002$$
$$+.0003$$
$$+.00005$$
$$+.000008$$
$$+.0000013$$

converges to a rational number.