24.2. Find the dual of each problem.

(i) Maximize \( f = 5x - y + 2z \)
subject to \( 7x + 2y - z \leq 8 \)
\( x - 3y - 2z \geq 4 \)
\( 3x - y + 6z = 5 \)
and \( x \geq 0, \ y \geq 0, \ z \geq 0 \).

(ii) Minimize \( g = 2r + 5s + t \)
subject to \( 9r - 3s + 5t \geq 7 \)
\( 6r - 4s - 3t \leq 2 \)
and \( r \leq 0, \ s \leq 0, \ t \leq 0 \).

(i) First multiply the second inequality by \(-1\) to change its sense to \(\leq\).

(ii) First multiply the second inequality by \(-1\) to change its sense to \(\geq\).

Multiply (i) by \(-3/2\) and add the result to the first row to obtain

\[
0 \quad 0 \quad 1 \quad -1/6 \quad -3/8 \quad 37/8
\]

Multiply (i) by \(1/4\) and add the result to the second row to obtain

\[
1 \quad 0 \quad 0 \quad 3/16 \quad 1/16 \quad 17/16
\]

Multiply (i) by 7 and add the result to the last row to obtain

\[
0 \quad 0 \quad 0 \quad 1/4 \quad 7/4 \quad 39/4
\]

Thus the calculated tableau is

\[
\begin{array}{cccccc}
1 & 2 & 5 & 4 & 1 & 2 & 3 & 2 \\
2 & -1 & 2 & 3 & 2 & -1 & 5 & 6 \\
3 & 5 & 1 & 1 & 5 & 2 & 1 & 7 \\
2 & 6 & 7 & * & 4 & 3 & 1 & *
\end{array}
\]

where the third row is now labeled \(P_2\), the label of the preceding pivotal column.

Observe that this tableau is a terminal tableau; the optimum solution of the minimum problem
is \(Q = (0, 1/4, 7/4)\), the optimum solution of the maximum problem is \(P = (17/16, 1/4)\), and
\[\text{max } f = \text{min } g = 39/4.\]