70 Prove that the tangent function has the power series \((6.92)\), and find the corresponding series for \(z/\sin z\) and \(\ln(\tan z)/z\).

71 Find a relation between the numbers \(T_n(1)\) and the coefficients of \(1/\cos z\).

72 What is \(\sum_k (-1)^k \binom{n}{k}\), the row sum of Euler’s triangle with alternating signs?

73 Prove that, for all integers \(n \geq 1\),

\[
z \cot z = \frac{z}{2^n} \cot \frac{z}{2^n} - \frac{z}{2^n} \tan \frac{z}{2^n} + \sum_{k=1}^{2^n-1} \frac{z^n}{2^n} \left( \cot \frac{z + k\pi}{2^n} - \cot \frac{z - k\pi}{2^n} \right),
\]

and show that the limit of the kth summand is \(2z^2/(z^2 - k^2\pi^2)\) for fixed \(k\) as \(n \to \infty\).

74 Prove the following relation that connects Stirling numbers, Bernoulli numbers, and Catalan numbers:

\[
\sum_{k=0}^{n} \binom{n+k}{k} \left( \frac{2n}{n+k} \right) \frac{(-1)^k}{k+1} = B_n \left( \frac{2n}{n+1} \right) \frac{1}{n+1}.
\]

75 Show that the four chessboard pieces of the \(64 = 65\) paradox can also be reassembled to prove that \(64 = 63\).

76 A sequence defined by the recurrence

\[
A_1 = x, \quad A_2 = y, \quad A_n = A_{n-1} + A_{n-2}
\]

has \(A_m = 1000000\) for some \(m\). What positive integers \(x\) and \(y\) make \(m\) as large as possible?

77 The text describes a way to change a formula involving \(F_{n+k}\) to a formula that involves \(F_n\) and \(F_{n+1}\) only. Therefore it’s natural to wonder if two such “reduced” formulas can be equal when they aren’t identical in form. Let \(P(x, y)\) be a polynomial in \(x\) and \(y\) with integer coefficients. Find a necessary and sufficient condition that \(P(F_{n+1}, F_n) = 0\) for all \(n \geq 0\).

78 Explain how to add positive integers, working entirely in the Fibonacci number system.

79 Is it possible that a sequence \(\{A_n\}\) satisfying the Fibonacci recurrence \(A_n = A_{n-1} + A_{n-2}\) can contain no prime numbers, if \(A_0\) and \(A_1\) are relatively prime?