89 Develop a general theory of the solutions to the two-parameter recurrence

\[
\begin{align*}
\binom{n}{k} &= (\alpha n + \beta k + \gamma) \binom{n-1}{k} \\
&\quad + (\alpha' n + \beta' k + \gamma') \binom{n-1}{k-1} + [n=k=0], \quad \text{for } n, k \geq 0,
\end{align*}
\]

assuming that \( \binom{n}{k} = 0 \) when \( n < 0 \) or \( k < 0 \). (Binomial coefficients, Stirling numbers, Eulerian numbers, and the sequences of exercises 17 and 31 are special cases.) What special values \((\alpha, \beta, \gamma, \alpha', \beta', \gamma')\) yield “fundamental solutions” in terms of which the general solution can be expressed?