Using the simplex method we obtain the following sequence of tableaux:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>1</th>
<th>0</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>-80</td>
<td>-160</td>
<td>-200</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>1</th>
<th>-\frac{1}{2}</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>\frac{1}{2}</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>-80</td>
<td>-120</td>
<td>0</td>
<td>40</td>
<td>8,000</td>
<td></td>
</tr>
</tbody>
</table>

Thus the minimum value of \( g \) is 12,000 and occurs at the point (40, 20). (Note that labels were not necessary since we were only solving the minimum problem.)

24.16. Suppose, in the preceding problem, the production cost at mine A is $250 per day and at mine B is $150 per day. How should the production schedule be changed?

In this case, the objective function to minimize is \( g = 250x + 150y \) subject to the same set of feasible solutions, i.e. conditions. Evaluate \( g \) at the corner points of the set of feasible solutions:

\[
g(P) = g(0,100) = 15,000, \quad g(Q) = g(20,50) = 12,500, \quad g(R) = g(40,20) = 13,000, \quad g(S) = g(80,0) = 20,000
\]

The minimum value of \( g \) occurs at \( Q = (20,50) \). Thus the company should now keep mine A open 20 days and mine B open 50 days for a total cost of $12,500.

### Supplementary Problems

**Duality**

24.17. Find the dual of each problem.

(i) Maximize \( f = 5x - 2y + 3z \)

subject to \( 3x - 4y + 2z \leq 9 \)

\( 2x + 7y + 5z \leq 11 \)

and \( x \geq 0, \quad y \geq 0, \quad z \geq 0 \).

(ii) Minimize \( g = 12x + 3y + 7z \)

subject to \( 6x - 2y + 5z \geq 3 \)

\( 2x + 3y - 4z \geq -2 \)

\( 3x + 9y + z \geq 8 \)

and \( x \geq 0, \quad y \geq 0, \quad z \geq 0 \).

24.18. Find the dual of each problem.

(i) Maximize \( f = 3x + 2y - 2z \)

subject to \( 4x + 3y - 2z \leq 11 \)

\( 5x - 2y + 7z \geq 2 \)

\( x + y + 3z \leq 15 \)

and \( x \geq 0, \quad y \geq 0, \quad z \geq 0 \).

(ii) Minimize \( g = r + 7s + 2t \)

subject to \( 7r + 3s + 5t \geq 9 \)

\( 4r + 8s - t \leq 10 \)

and \( r \geq 0, \quad s \geq 0, \quad t \geq 0 \).

**Simplex Method**

24.19. Find the initial tableau and all possible pivot entries for each of the problems given in Problem 24.17.