taken as the faces of the grid model is more accurate than ordinary BP. We also found the TR SRG construction to be more accurate than GBP run on Loop-SRGs constructed from fundamental, but non-TR cycle bases on Ising grids.

8.2 Partially TR SRGs

The previous experiment considered a class of MNs for which a TR basis is known. This section considers more general MNs.

The algorithm in Section 7 seeks an initial TR subgraph $H$ of the graph $G$. The previous experiments showed that GBP yields accurate approximations when $H = G$. When a graph contains no TR core (i.e. $H = \emptyset$) the algorithm Construct Basis method simply finds a fundamental basis of $G$. We wish to study the performance of GBP between these two extremes - i.e. on partially TR SRGs. We conducted experiments on two types of MNs, where the size of $H$ (relative to $G$) can be controlled.

The following experiments include a comparison to the Iterative Join Graph Propagation (IJGP) method [3]. This method forms a join graph (i.e. a 1-connected and 1-balanced RG) using a heuristic, "mini-bucket" clustering strategy. In IJGP, the number of variables appearing in an outer region is restricted to be less than or equal to an $iBound$ parameter. The $iBound$ controls the computational complexity of message computations in IJGP because in the join graph construction each outer region forms a clique over all variables in that region. Importantly, since Loop-SRGs assume a loop structure in the outer regions, message computations on Loop-SRGs are equivalent to IJGP with $iBound = 3$.

8.2.1 Partial $K$-Trees

In these experiments we construct a set of partial $K$-tree instances via the following procedure. We first build a random $K$-tree on $n$ vertices using the process described in [6]. The number of neighbors (degree) of the vertices in $K$-trees constructed by this procedure follows a power law. This means there will exist a few vertices that are adjacent to most of the vertices in $G$. As a result, the TR core will comprise a large portion of $G$. To reduce the size of the TR core, we iteratively remove edges from the $K$-tree as follows: choose a vertex $v$ with probability proportional to the current degree of that vertex. Modify $G$ by removing an edge from $v$ to one of its neighbors, so long as removing that edge does not disconnect $G$. This process is repeated until the ratio of the maximum degree in $G$ to $n$ falls below some threshold. We refer to this ratio as the connectivity of the graph. A random MN is formed over each partial $K$-tree structure by assigning random unary and pairwise potentials to the vertices and edges.

Figure 5 shows the performance of GBP as a function of connectivity. In these figures, we generated $M = 100$ random MN instances at each level of connectivity (with $K = 10$, $n = 100$, $\sigma_h = 1$ and $\sigma_{w_{ij}} = 0.3$). The partially TR SRGs are found by choosing the TR core to be the subgraph $H$ found using the max degree vertex as the root of the star construction described in Section 6. Cycles are added to this TR core as described in algorithm Construct Basis. The partially TR SRGs are compared to Loop-SRGs formed by finding a fundamental cycle basis (FCB) of each partial $K$-tree (constructed using algorithm “Construct Basis” with $H = \emptyset$). Importantly, these FCBs do not build upon the TR core. Figure 5 shows that the benefit of the partially TR SRG diminishes as connectivity is decreased. This behavior confirms our belief that the benefit of finding a TR core decreases as the TR core comprises a smaller proportion of cycles in the fundamental basis. Even so, it is important to note that choosing a Loop-SRG with outer regions forming a FCB yields more accurate approximations than both IJGP and BP.

\footnote{K-trees are chordal graphs w/ size $K$ maximal cliques. Partial $K$-trees are non-chordal graphs w/ size $K$ maximal cliques}