cycles in the basis, 81 of which come from the TR core).

(a) Error $L_1$ on grids with long range interactions

(b) Error $Z$ on grids with long range interactions

Figure 7: Performance of the different SRG constructions as an increasing number of long range interactions are added to a $10 \times 10$ grid.

9 Conclusion

This paper provides some new guidance in the construction of region graphs. In particular, we connected the problem of choosing the loop outer regions of a Loop-SRG to that of finding a fundamental cycle basis of the undirected graph describing a MN. We proposed tree robustness as a refinement to the criterion that the loop regions form a fundamental basis and offered a characterization of the class of TR cycle bases. We identified TR cycle bases for planar and complete graphs. This characterization helps explain the success of GBP on the “star” construction of [31] for complete graphs and the “all faces” construction on planar graphs. We also proposed a practical Loop-SRG construction that first identifies a TR core and then expands to a full cycle basis using an ear construction which makes sure the final basis is still fundamental (and partially TR).

The experiments in this paper confirm that GBP can yield very accurate approximations when the loop regions of a Loop-SRG form a fundamental basis and that these approximations can be further improved by choosing a fundamental basis that is at least partially tree robust. The criteria proposed in this paper also lead to approximations that are comparable to IJGP run with a much higher $iBound$ (and therefore much higher space and computational complexity).

These findings open the door for much future work. Rather than simply finding a TR subgraph, as the method in Section 7 does, it would be preferable to have an algorithm that searches for TR bases in a graph or at the very least identifies when a graph does not admit a TR basis. The recommendations in this paper are also purely structural in nature. A natural extension would be to incorporate interaction strengths in the search for suitable loop regions.

The current paper considered pairwise interactions only. A natural extension is to consider factor graphs or more generally region graphs. We argue that we should be looking for a maximal collection of subsets $\{C_i\}$ of outer regions (or factors) that have the property that there exists an ordering $\pi$ for which $C_{\pi(i)} \setminus \{C_{\pi(1)} \cup \cdots \cup C_{\pi(i-1)}\} \neq \emptyset$. In other words, the subsets can be ordered so that subset $C_{\pi(i)}$ has some element that does not appear in any subset preceding it in the ordering. This definition is identical to the one for a fundamental cycle basis (see definition 6) and will guarantee through the reduction rules of [31] that the region graph (factor graph) can be decomposed into independent variable nodes, establishing non-singularity. A next step would then be to define tree-robustness as a collection of region-subsets that can still be decomposed along some ordering if we do not allow certain elements that correspond to the regions of any embedded junction tree to become unique. Again this is very similar to definitions 7 and 8 for TR cycle bases. Whether these generalizations can be captured with mathematical structures as elegant as the theory of cycle spaces (or more generally matroids) remains to be seen and will be left for future research.

Acknowledgements

This material is based upon work supported by the National Science Foundation under Grant No. 0914783, 0928427, 1018433.

References
