Obviously \( P_n = 1 \) for all \( n \geq 0 \). And a little thought proves that we have \( N_n = \lfloor n/5 \rfloor + 1 \): To make \( n \) cents out of pennies and nickels, we must choose either 0 or 1 or \ldots or \( \lfloor n/5 \rfloor \) nickels, after which there's only one way to supply the requisite number of pennies. Thus \( P_n \) and \( N_n \) are simple; but the values of \( D_n \), \( Q_n \), and \( C_n \) are increasingly more complicated.

One way to deal with these formulas is to realize that \( 1 + z^n + z^{2n} + \ldots \) is just \( 1/(1 - z^n) \). Thus we can write

\[
\begin{align*}
P & = 1/(1 - z), \\
N & = P/(1 - z^2), \\
D & = N/(1 - z^{10}), \\
Q & = D/(1 - z^{25}), \\
C & = Q/(1 - z^{50}).
\end{align*}
\]

Multiplying by the denominators, we have

\[
\begin{align*}
(1 - z)P & = 1, \\
(1 - z^2)N & = P, \\
(1 - z^{10})D & = N, \\
(1 - z^{25})Q & = D, \\
(1 - z^{50})C & = Q.
\end{align*}
\]

Now we can equate coefficients of \( z^n \) in these equations, getting recurrence relations from which the desired coefficients can quickly be computed:

\[
\begin{align*}
P_n & = P_{n-1} + \lfloor n=0 \rfloor, \\
N_n & = N_{n-5} + P_n, \\
D_n & = D_{n-10} + N_n, \\
Q_n & = Q_{n-25} + D_n, \\
C_n & = Q_n - Q_{n-25}.
\end{align*}
\]

For example, the coefficient of \( z^n \) in \( D = (1 + z^{25})Q \) is equal to \( Q_n - Q_{n-25} \); so we must have \( Q_n = Q_{n-25} = D_n \), as claimed.

We could unfold these recurrences and find, for example, that \( Q_n = D_n + D_{n-25} + D_{n-50} + D_{n-75} + \ldots \), stopping when the subscripts get negative. But the non-iterated form is convenient because each coefficient is computed with just one addition, as in Pascal’s triangle.

Let's use the recurrences to find \( C_{50} \). First, \( C_{50} = C_0 + Q_{50} \); so we want to know \( Q_{50} \). Then \( Q_{50} = Q_{25} + D_{50} \), and \( Q_{25} = Q_0 + D_{25} \); so we also want to know \( D_{50} \) and \( D_{25} \). These \( D_n \) depend in turn on \( D_{40} \), \( D_{30} \), \( D_{20} \), \( D_{15} \), \( D_{10} \), \( D_5 \), and on \( N_{50} \), \( N_{45} \), \ldots, \( N_5 \). A simple calculation therefore suffices to