Thus the equation \( y = h(t) \) has the graph shown in Figure 6.3.3. This function can be thought of as a rectangular pulse.

\[
\text{FIGURE 6.3.3} \quad \text{Graph of } y = u_{\pi}(t) - u_{2\pi}(t).
\]

The Laplace transform of \( u_c \) is easily determined:

\[
\mathcal{L}\{u_c(t)\} = \int_0^\infty e^{-st} u_c(t) \, dt = \int_c^\infty e^{-st} \, dt = \frac{e^{-cs}}{s}, \quad s > 0.
\]

(2)

For a given function \( f \), defined for \( t \geq 0 \), we will often want to consider the related function \( g \) defined by

\[
y = g(t) = \begin{cases} 
 0, & t < c, \\
 0, & t \geq c,
\end{cases} \quad f(t - c),
\]

which represents a translation of \( f \) a distance \( c \) in the positive \( t \) direction; see Figure 6.3.4. In terms of the unit step function we can write \( g(t) \) in the convenient form

\[
g(t) = u_c(t) f(t - c).
\]

The unit step function is particularly important in transform use because of the following relation between the transform of \( f(t) \) and that of its translation \( u_c(t) f(t - c) \).

\[
\text{Theorem 6.3.1} \quad \text{If } F(s) = \mathcal{L}\{f(t)\} \text{ exists for } s > a \geq 0, \text{ and if } c \text{ is a positive constant, then}
\]

\[
\mathcal{L}\{u_c(t) f(t - c)\} = e^{-cs} \mathcal{L}\{f(t)\} = e^{-cs} F(s), \quad s > a.
\]

(3)