Find the inverse transform of

\[ F(s) = \frac{1 - e^{-2s}}{s^2}. \]

From the linearity of the inverse transform we have

\[ f(t) = \mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{ \frac{1}{s^2} \right\} - \mathcal{L}^{-1}\left\{ \frac{e^{-2s}}{s^2} \right\} = t - u_2(t)(t - 2). \]

The function \( f \) may also be written as

\[ f(t) = \begin{cases} t, & 0 \leq t < 2, \\ 2, & t \geq 2. \end{cases} \]

The following theorem contains another very useful property of Laplace transforms that is somewhat analogous to that given in Theorem 6.3.1.

**Theorem 6.3.2** If \( F(s) = \mathcal{L}\{f(t)\} \) exists for \( s > a \geq 0 \), and if \( c \) is a constant, then

\[ \mathcal{L}\{e^{ct}f(t)\} = F(s - c), \quad s > a + c. \tag{5} \]

Conversely, if \( f(t) = \mathcal{L}^{-1}\{F(s)\} \), then

\[ e^{ct}f(t) = \mathcal{L}^{-1}\{F(s - c)\}. \tag{6} \]

According to Theorem 6.3.2, multiplication of \( f(t) \) by \( e^{ct} \) results in a translation of the transform \( F(s) \) a distance \( c \) in the positive \( s \) direction, and conversely. The proof of this theorem requires merely the evaluation of \( \mathcal{L}\{e^{ct}f(t)\} \). Thus

\[
\mathcal{L}\{e^{ct}f(t)\} = \int_0^\infty e^{-st}e^{ct}f(t)\,dt = \int_0^\infty e^{-(s-c)t}f(t)\,dt = F(s-c),
\]