Table 321 Simple sequences and their generating functions.

<table>
<thead>
<tr>
<th>sequence</th>
<th>generating function</th>
<th>closed form</th>
</tr>
</thead>
<tbody>
<tr>
<td>((1,0,0,0,0,0,\ldots))</td>
<td>(\sum_{n\geq0} [n=0] z^n)</td>
<td>1</td>
</tr>
<tr>
<td>((0,\ldots,0,1,0,0,\ldots))</td>
<td>(\sum_{n\geq0} [n=m] z^n)</td>
<td>(z^m)</td>
</tr>
<tr>
<td>((1,1,1,1,1,\ldots))</td>
<td>(\sum_{n\geq0} z^n)</td>
<td>(1 \pm z)</td>
</tr>
<tr>
<td>((1,1,1,1,1,\ldots))</td>
<td>(\sum_{n\geq0} (-1)^n z^n)</td>
<td>(1 \pm z)</td>
</tr>
<tr>
<td>((1,0,1,0,1,0,\ldots))</td>
<td>(\sum_{n\geq0} [2\backslash n] z^n)</td>
<td>(1 \pm z^2)</td>
</tr>
<tr>
<td>((1,0,\ldots,0,1,0,\ldots,0,1,0,\ldots))</td>
<td>(\sum_{n\geq0} [m\backslash n] z^n)</td>
<td>(1 \pm z^m)</td>
</tr>
<tr>
<td>((1,2,3,4,5,6,\ldots))</td>
<td>(\sum_{n\geq0} (n+1) z^n)</td>
<td>(1 \pm (1 - z)^2)</td>
</tr>
<tr>
<td>((1,2,4,8,16,32,\ldots))</td>
<td>(\sum_{n\geq0} 2^n z^n)</td>
<td>(1 \pm 2z)</td>
</tr>
<tr>
<td>((1,4,6,4,1,0,0,\ldots))</td>
<td>(\sum_{n\geq0} 4 \binom{n}{1} z^n)</td>
<td>((1 + z)^4)</td>
</tr>
<tr>
<td>((1,c,(\frac{c}{2}), (\frac{c}{3}),\ldots))</td>
<td>(\sum_{n\geq0} \binom{c}{n} z^n)</td>
<td>((1 + z)^c)</td>
</tr>
<tr>
<td>((1,c,(\frac{c+1}{2}), (\frac{c+2}{3}),\ldots))</td>
<td>(\sum_{n\geq0} \binom{c+n-1}{n} z^n)</td>
<td>(1 \pm \frac{1}{(1 - z)^c})</td>
</tr>
<tr>
<td>((1,c,c^2,c^3,\ldots))</td>
<td>(\sum_{n\geq0} c^n z^n)</td>
<td>(1 \pm cz)</td>
</tr>
<tr>
<td>((1,(\frac{m+1}{m}), (\frac{m+2}{m}), (\frac{m+3}{m}),\ldots))</td>
<td>(\sum_{n\geq0} \binom{m+n}{m} z^n)</td>
<td>(1 \pm \frac{1}{(1 - z)^{m+1}})</td>
</tr>
<tr>
<td>((0,1,\frac{1}{2},\frac{1}{3},\frac{1}{4},\ldots))</td>
<td>(\sum_{n\geq1} \frac{1}{n} z^n)</td>
<td>(\ln\frac{1}{1 - z})</td>
</tr>
<tr>
<td>((0,1,-\frac{1}{2},\frac{1}{3},-\frac{1}{4},\ldots))</td>
<td>(\sum_{n\geq1} \frac{(-1)^{n+1}}{n} z^n)</td>
<td>(\ln(1 + z))</td>
</tr>
<tr>
<td>((1,1,\frac{1}{2},\frac{1}{6},\frac{1}{24},\frac{1}{120},\ldots))</td>
<td>(\sum_{n\geq0} \frac{1}{n!} z^n)</td>
<td>(e^z)</td>
</tr>
</tbody>
</table>

Hint: If the sequence consists of binomial coefficients, its generating function usually involves a binomial, \(1 \pm z\).