standard semantics but not according to the new semantics. Discuss which outcome makes more intuitive sense for your examples.

8.8 Does the fact $\text{Spouse}(\text{George}, \text{Laura})$ follow from the facts $\text{Jim} \text{ George}$ and $\text{Spouse}(\text{Jim}, \text{Laura})$? If so, give a proof; if not, supply additional axioms as needed. What happens if we use $\text{Spouse}$ as a unary function symbol instead of a binary predicate?

8.9 This exercise uses the function $\text{Map Color}$ and predicates $\text{In}(x, y)$, $\text{Borders}(x, y)$, and $\text{Country}(x)$, whose arguments are geographical regions, along with constant symbols for various regions. In each of the following we give an English sentence and a number of candidate logical expressions. For each of the logical expressions, state whether it (1) correctly expresses the English sentence; (2) is syntactically invalid and therefore meaningless; or (3) is syntactically valid but does not express the meaning of the English sentence.

a. Paris and Marseilles are both in France.
   (i) $\text{In}(\text{Paris} \land \text{Marseilles}, \text{France})$.
   (ii) $\text{In}(\text{Paris}, \text{France}) \land \text{In}(\text{Marseilles}, \text{France})$.
   (iii) $\text{In}(\text{Paris}, \text{France}) \lor \text{In}(\text{Marseilles}, \text{France})$.

b. There is a country that borders both Iraq and Pakistan.
   (i) $\exists c \text{ Country}(c) \land \text{Border}(c, \text{Iraq}) \land \text{Border}(c, \text{Pakistan})$.
   (ii) $\exists c \text{ Country}(c) \land \text{Border}(c, \text{Iraq}) \land \text{Border}(c, \text{Pakistan})$.
   (iii) $\exists c \text{ Country}(c) \land [\text{Border}(c, \text{Iraq}) \land \text{Border}(c, \text{Pakistan})]$.
   (iv) $\forall c \text{ Border}(\text{Country}(c), \text{Iraq} \land \text{Pakistan})$.

c. All countries that border Ecuador are in South America.
   (i) $\forall c \text{ Country}(c) \land \text{Border}(c, \text{Ecuador}) = \text{In}(c, \text{SouthAmerica})$.
   (ii) $\forall c \text{ Country}(c) \land \text{Border}(c, \text{Ecuador}) = \text{In}(c, \text{SouthAmerica})$.
   (iii) $\forall c \text{ Country}(c) \land \text{Border}(c, \text{Ecuador}) = \text{In}(c, \text{SouthAmerica})$.
   (iv) $\forall c \text{ Border}(\text{Country}(c), \text{Ecuador}) = \text{In}(c, \text{SouthAmerica})$.

d. No region in South America borders any region in Europe.
   (i) $\neg [c, d \text{ In}(c, \text{SouthAmerica}) \land \text{In}(d, \text{Europe}) \land \text{Borders}(c, d)]$.
   (ii) $\forall c, d [\text{In}(c, \text{SouthAmerica}) \land \text{In}(d, \text{Europe})] \Rightarrow \neg \text{Borders}(c, d)$.
   (iii) $\forall c \text{ In}(c, \text{SouthAmerica}) = \text{In}(d, \text{Europe}) \land \neg \text{Borders}(c, d)$.
   (iv) $\forall c \text{ In}(c, \text{SouthAmerica}) \land \text{In}(d, \text{Europe}) \Rightarrow \neg \text{Borders}(c, d)$.

e. No two adjacent countries have the same map color.
   (i) $\forall x, y \text{ Country}(x) \land \text{Country}(y) \land \text{Borders}(x, y) \land \neg (\text{MapColor}(x) = \text{MapColor}(y))$.
   (ii) $\forall x, y \text{ Country}(x) \land \text{Country}(y) \land \text{Borders}(x, y) \land \neg (\text{MapColor}(x) = \text{MapColor}(y))$.
   (iii) $\forall x, y \text{ Country}(x) \land \text{Country}(y) \land \text{Borders}(x, y) \land \neg (\text{MapColor}(x) = \text{MapColor}(y))$.
   (iv) $\forall x, y \text{ Country}(x) \land \text{Country}(y) \land \text{Borders}(x, y) \land \text{MapColor}(x, y)$.
8.10 Consider a vocabulary with the following symbols:

- **Occupation** \((p, o)\): Predicate. Person \(p\) has occupation \(o\).
- **Customer** \((p_1, p_2)\): Predicate. Person \(p_1\) is a customer of person \(p_2\).
- **Boss** \((p_1, p_2)\): Predicate. Person \(p_1\) is a boss of person \(p_2\).
- **Doctor, Surgeon, Lawyer, Actor**: Constants denoting occupations.
- **Emily, The**: Constants denoting people.

Use these symbols to write the following assertions in first-order logic:

a. Emily is either a surgeon or a lawyer.

b. Joe is an actor, but he also holds another job,

c. All surgeons are doctors.

d. Joe does not have a lawyer (i.e., is not a customer of any lawyer).

e. Emily has a boss who is a lawyer.

f. There exists a lawyer all of whose customers are doctors.

g. Every surgeon has a lawyer.

8.11 Complete the following exercises about logical sentences:

a. Translate into good, natural English (no **xs** or **ys**):

\[ \forall x, y, l \text{ SpeaksLanguage}(x, l) \land \text{SpeaksLanguage}(y, l) \land \text{Understands}(x, y) \]

h. Explain why this sentence is entailed by the sentence.

\[ \forall x, y, l \text{ SpeaksLanguage}(x, l) \land \text{SpeaksLanguage}(y, l) \land \text{Understands}(x, y) \]

c. Translate into first-order logic the following sentences:

(i) Understanding leads to friendship.

(ii) Friendship is transitive.

Remember to define all predicates, functions, and constants you use

8.12 Rewrite the first two Peano axioms in Section 8.3.3 as a single axiom that defines

\[ \text{NatNum}(x) \]

so as to exclude the possibility of natural numbers except for those generated by the successor function.

8.13 Equation (8.4) on page 306 defines the conditions under which a square is breezy. Here we consider two other ways to describe this aspect of the *wumpus* world.

**Diagnostic Rule**

a. We can write **diagnostic rules** leading from observed effects to hidden causes. For finding pits, the obvious diagnostic rules say that if a square is breezy, some adjacent square must contain a pit; and if a square is not breezy, then no adjacent square contains a pit. Write these two rules in first-order logic and show that their conjunction is logically equivalent to Equation (8.4).

**Causal Rule**

b. We can write **causal rules** leading from cause to effect. One obvious causal rule is that a pit causes all adjacent squares to be breezy. Write this rule in first-order logic, explain why it is incomplete compared to Equation (8.4), and supply the missing axiom.