Observe that we have used Theorem 6.3.1 to write the inverse transforms of $e^{-5s}H(s)$ and $e^{-20s}H(s)$, respectively. Finally, to determine $h(t)$ we use the partial fraction expansion of $H(s)$:

$$H(s) = \frac{a}{s} + \frac{bs + c}{2s^2 + s + 2}. \quad (8)$$

Upon determining the coefficients we find that $a = \frac{1}{2}$, $b = -1$, and $c = -\frac{1}{2}$. Thus

$$H(s) = \frac{1/2}{s} - \frac{s + \frac{1}{2}}{2s^2 + s + 2} = \frac{1/2}{s} - \left(\frac{1}{2}\right)\left(\frac{s + \frac{1}{2}}{(s + \frac{1}{2})^2 + \frac{15}{16}}\right). \quad (9)$$

so that, by referring to lines 9 and 10 of Table 6.2.1, we obtain

$$h(t) = \frac{1}{2} - \frac{1}{2}\left[e^{-t/4}\cos(\sqrt{15} t/4) + (\sqrt{15}/15)e^{-t/4}\sin(\sqrt{15} t/4)\right]. \quad (10)$$

In Figure 6.4.1 the graph of $y = \phi(t)$ from Eqs. (7) and (10) shows that the solution consists of three distinct parts. For $0 < t < 5$ the differential equation is

$$2y'' + y' + 2y = 0 \quad (11)$$

and the initial conditions are given by Eq. (3). Since the initial conditions impart no energy to the system, and since there is no external forcing, the system remains at rest; that is, $y = 0$ for $0 < t < 5$. This can be confirmed by solving Eq. (11) subject to the initial conditions (3). In particular, evaluating the solution and its derivative at $t = 5$, or more precisely, as $t$ approaches 5 from below, we have

$$y(5) = 0, \quad y'(5) = 0. \quad (12)$$

Once $t > 5$, the differential equation becomes

$$2y'' + y' + 2y = 1, \quad (13)$$

whose solution is the sum of a constant (the response to the constant forcing function) and a damped oscillation (the solution of the corresponding homogeneous equation). The plot in Figure 6.4.1 shows this behavior clearly for the interval $5 \leq t \leq 20$. An expression for this portion of the solution can be found by solving the differential equation (13) subject to the initial conditions (12). Finally, for $t > 20$ the differential equation becomes Eq. (11) again, and the initial conditions are obtained by evaluating the solution of Eqs. (13), (12) and its derivative at $t = 20$. These values are

$$y(20) \approx 0.50162, \quad y'(20) \approx 0.01125. \quad (14)$$

The initial value problem (11), (14) contains no external forcing, so its solution is a damped oscillation about $y = 0$, as can be seen in Figure 6.4.1.