8.14 Write axioms describing the predicates Grandchild, Great grandparent, Ancestor, Brother, Sister, Daughter, Sort, First Cousin, Brother In Law, Sister In Law, Aunt, and Uncle. Find out the proper definition of nth cousin n times removed, and write the definition in first-order logic. Now write down the basic facts depicted in the family tree in Figure 8.7. Using a suitable logical reasoning system, TELL it all the sentences you have written down, and ASK it who are Elizabeth's grandchildren, Diana's brothers-in-law, Zara's great-grandparents, and Eugenie's ancestors.

8.15 Explain what is wrong with the following proposed definition of the set membership predicate E:
\[ \forall x, s \ (x \in \{x \cap s\}) \]
\[ \forall x, s \ (x \in s) \]

If using the set axioms as examples, write axioms for the list domain, including all the constants, functions, and predicates mentioned in the chapter.

8.17 Explain what is wrong with the following proposed definition of adjacent squares in the wumpus world:
\[ \forall a', b\ ADJACENT([x, y], [x+1, y]) \land ADJACENT([x, y], [x, y+1]) \]

8.18 Write out the axioms required for reasoning about the wumpus's location, using a constant symbol Wumpus and a binary predicate At(Wumpus, Location). Remember that there is only one wumpus.

8.19 Assuming predicates Parent(p, q) and Female(p) and constants Joan and Kevin, with the obvious meanings, express each of the following sentences in first-order logic. (You may use the abbreviation \( \exists! \) to mean "there exists exactly one.")
   a. Joan has a daughter (possibly more than one, and possibly sons as well).
   b. Joan has exactly one daughter (but may have sons as well).
   c. Joan has exactly one child, a daughter.
   d. Joan and Kevin have exactly one child together.
   e. Joan has at least one child with Kevin, and no children with anyone else.
8.20 Arithmetic assertions can be written in first-order logic with the predicate symbol $<$, the function symbols $+$ and $x$, and the constant symbols 0 and 1. Additional predicates can also be defined with biconditionals.

a. Represent the property "x is an even number."

b. Represent the property "x is prime."

c. Goldbach's conjecture is the conjecture (unproven as yet) that every even number is equal to the sum of two primes. Represent this conjecture as a logical sentence.

8.21 In Chapter 6, we used equality to indicate the relation between a variable and its value. For instance, we wrote $WA = red$ to mean that Western Australia is colored red. Representing this in first-order logic, we must write more verbosely $\text{ColorOf}(WA) = \text{red}$. What incorrect inference could be drawn if we wrote sentences such as $WA = \text{red}$ directly as logical assertions?

8.22 Write in first-order logic the assertion that every key and at least one of every pair of socks will eventually be lost forever, using only the following vocabulary: $\text{Key}(x)$, x is a key; $\text{Sock}(x)$, x is a sock; $\text{Pair}(x, y)$, r and y are a pair; $\text{Now}$, the current time; $\text{Before}(t_1, t_2)$, time $t_1$ comes before time $t_2$; $\text{Lost}(x, t)$, object x is lost at time t.

8.23 For each of the following sentences in English, decide if the accompanying first-order logic sentence is a good translation. If not, explain why not and correct it. (Some sentences may have more than one error!)

a. No two people have the same social security number,

$b, y, n \quad \text{Person}(b) \land \text{Person}(y) \land [\text{HasSS}(b, n) \land \text{HasSS}(y, n)].$

b. John's social security number is the same as Mary's.

$n \quad \text{HasSS}(\text{John}, n) \land \text{HasSS}(\text{Mary}, n).$

c. Everyone's social security number has nine digits.

$\forall x, n \quad \text{Person}(x) \quad [\text{HasSS}(x, n) \land \text{Digits}(n, 9)].$

d. Rewrite each of the above (uncorrected) sentences using a function symbol $\text{SS}$# instead of the predicate $\text{HasSS}$#.

8.24 Represent the following sentences in first-order logic, using a consistent vocabulary (which you must define);


b. Every student who takes French passes it.

c. Only one student took Greek in spring 2001.

d. The best score in Greek is always higher than the best score in French.

e. Every person who buys a policy is smart.

f. No person buys an expensive policy.

g. There is an agent who sells policies only to people who are not insured.