25.20. Find a solution to each game.

\[
\begin{array}{ccc}
2 & -4 & -3 \\
-1 & -2 & 1 \\
0 & 1 & 1 \\
\end{array}
\quad
\begin{array}{ccc}
-2 & 1 & 2 \\
3 & -1 & -2 \\
-1 & 1 & 3 \\
\end{array}
\]

25.21. Consider the game

\[
\begin{array}{cc}
4 & 1 \\
3 & a \\
\end{array}
\]

Find a solution to the game if (i) \( a < 1 \), (ii) \( 1 < a < 3 \), (iii) \( a > 3 \).

25.22. Each of two players \( R \) and \( C \) has a dime and a quarter. They each show a coin simultaneously. If the coins are the same, \( R \) wins \( C \)'s coin; if the coins are different, \( C \) wins \( R \)'s coin. Represent the game as a matrix game and find a solution.

25.23. Each of two players \( R \) and \( C \) has a penny, nickel and dime. They each show a coin simultaneously. If the total amount of money shown is even, \( R \) wins \( C \)'s coin; if it is odd, \( C \) wins \( R \)'s coin. Represent the game as a matrix game and find a solution.

**THEOREMS**

25.24. Suppose every entry in a matrix game \( A \) is increased by an amount \( k \). Show that the value of the game also increases by \( k \), but that the optimum strategies remain the same.

25.25. Show that if every entry in a matrix game is positive, then the value of the game is positive.

**Answers to Supplementary Problems**

25.17. (i) \( p^0 = (0, 1) \), \( q^0 = (0, 1) \), \( v = 1 \); (ii) \( p^0 = (1, 0) \), \( q^0 = (0, 1) \), \( v = 2 \); (iii) \( p^0 \) can be any strategy, \( q^0 = (1, 0) \), \( v = 1 \).

25.18. (i) \( p^0 = (7/3, 5/3) \), \( q^0 = (4/3, 4/3) \), \( v = 2 \); (ii) \( p^0 = (3/4, 1/4) \), \( q^0 = (5/12, 7/12) \), \( v = 1/4 \); (iii) \( p^0 = (7/8, 1/8) \), \( q^0 = (5/8, 3/8) \), \( v = 3/8 \).

25.19. (i) \( p^0 = (5/8, 3/8) \), \( q^0 = (3/8, 5/8) \), \( v = 1/8 \); (ii) \( p^0 = (5/7, 2/7) \), \( q^0 = (5/7, 2/7) \), \( v = 3/7 \); (iii) \( p^0 = (2/7, 5/7) \), \( q^0 = (4/7, 3/7) \), \( v = 13/7 \).

25.20. (i) \( p^0 = (1/7, 0, 6/7) \), \( q^0 = (5/7, 2/7) \), \( v = 2/7 \); (ii) \( p^0 = (0, 1/3, 2/3) \), \( q^0 = (1/3, 2/3) \), \( v = 1/3 \).

25.21. (i) \( p^0 = (1, 0) \), \( q^0 = (0, 1) \), \( v = 1 \); (ii) \( p^0 = (0, 1) \), \( q^0 = (0, 1) \), \( v = a \); (iii) \( p^0 = \left( \frac{a - 3}{a}, \frac{a}{b} \right) \), \( q^0 = \left( \frac{a - 1}{a}, \frac{1}{b} \right) \), \( v = (4a - 3)/a \).

25.22. 

\[
\begin{array}{cc}
10 & -10 \\
-25 & 25 \\
\end{array}
\]

; \( p^0 = (5/7, 2/7) \), \( q^0 = (1/2, 1/2) \), \( v = 0 \).

25.23. 

\[
\begin{array}{ccc}
1 & 5 & 10 \\
5 & 1 & 5 \\
10 & -10 & 10 \\
\end{array}
\]

; \( p^0 = (10/11, 0, 1/11) \), \( q^0 = (1/2, 0, 1/2) \), \( v = 0 \).