Fortunately the problem is easy to fix, since we can add \([n = 1]\) to the right; this adds 1 when \(n = 1\), and it makes no change when \(n \neq 1\). So, we have

\[
g_n = g_{n-1} + g_{n-2} + [n=1];
\]

this is the equation called for in Step 1.

Step 2 now asks us to transform the equation for \(\{g_n\}\) into an equation for \(G(z) = \sum_n g_n z^n\). The task is not difficult:

\[
G(z) = \sum_n g_n z^n = \sum_n g_{n-1} z^n + \sum_n g_{n-2} z^n + \sum_n [n = 1]z^n
\]

\[
= \sum_n g_n z^{n+1} + \sum_n g_n z^{n+2} + z
\]

\[
= zG(z) + z^2G(z) + z.
\]

Step 3 is also simple in this case; we have

\[
G(z) = \frac{z}{1 - z - z^2},
\]

which of course comes as no surprise.

Step 4 is the clincher. We carried it out in Chapter 6 by having a sudden flash of inspiration; let’s go more slowly now, so that we can get through Step 4 safely later, when we meet problems that are more difficult. What is

\[
[z^n] \frac{z}{1 - z - z^2};
\]

the coefficient of \(z^n\) when \(z/(1 - z - z^2)\) is expanded in a power series? More generally, if we are given any rational function

\[
R(z) = \frac{P(z)}{Q(z)},
\]

where \(P\) and \(Q\) are polynomials, what is the coefficient \([z^n] R(z)\)?

There’s one kind of rational function whose coefficients are particularly nice, namely

\[
\frac{a}{(1 - \rho z)^{m+1}} = \sum_{n \geq 0} \binom{m+n}{m} \rho^n z^n \quad (7.25)
\]

(The case \(\rho = 1\) appears in Table 321, and we can get the general formula shown here by substituting \(\rho z\) for \(z\).) A finite sum of functions like \((7.25)\),

\[
S(z) = \frac{a_1}{(1 - \rho_1 z)^{m_1+1}} + \frac{a_2}{(1 - \rho_2 z)^{m_2+1}} + \cdots + \frac{a_l}{(1 - \rho_l z)^{m_l+1}}, \quad (7.26)
\]