\(\text{reg}_{M,P^+}(fEg)\)
\[= \sup_{Pr \in P} (\alpha_{Pr} \sum_{s \in S} Pr(s) \text{reg}_M(fEg,s))\]
\[= \sup_{Pr \in P} (\alpha_{Pr} (\sum_{s \in E} Pr(s) \text{reg}_M(f,s)) + \sum_{s \in E^c} Pr(s) \text{reg}_M(g,s))\]
\[= \sup_{Pr \in P} (\alpha_{Pr} \sum_{s \in S} Pr(s) \text{reg}_M(g,s))\]
\[= \text{reg}_{M,P^+}(g).\]

Thus, \(fEg \sim_M g\) for all acts \(f, g\) and menus \(M\) containing \(fEg\) and \(g\), which means that \(E\) is null.

For the second part, we first show that if \(\overline{P}^+(E) > 0\), then for all \(f, h \in P\), we have that
\[\text{reg}_{M,Eh,P^+}(f Eh) = \overline{P}^+(E)\text{reg}_{M,P^+|E}(f).\]

We proceed as follows:
\[\text{reg}_{M,Eh,P^+}(f Eh)\]
\[= \sup_{Pr \in P} (\alpha_{Pr} \sum_{s \in S} Pr(s) \text{reg}_{M,Eh}(f Eh))\]
\[= \sup_{Pr \in P} (\alpha_{Pr} \sum_{s \in S} Pr(s) \text{reg}_{M}(f,s) + \sum_{s \in E^c} Pr(s) \text{reg}_{M}(g,s))\]
\[= \sup_{Pr \in P} (\overline{P}^+(E) \alpha_{Pr} \sum_{s \in S} Pr(s) \text{reg}_{M}(f,s) + \sum_{s \in E^c} Pr(s) \text{reg}_{M}(g,s))\]
\[= \overline{P}^+(E) \cdot \text{reg}_{M,P^+|E}(f).\]

Thus, for all \(h \in M\),
\[\text{reg}_{M,Eh,P^+}(f Eh) \leq \text{reg}_{M,Eh,P^+}(g Eh)\]
iff \(\overline{P}^+(E) \cdot \text{reg}_{M,P^+|E}(f) \leq \overline{P}^+(E) \cdot \text{reg}_{M,P^+|E}(g)\)
iff \(\text{reg}_{M,P^+|E}(f) \leq \text{reg}_{M,P^+|E}(g)\).

It follows that the order induced by \(Pr^+\) satisfies MDC.

Moreover, if 1–10 and MDC hold, then for the weighted set \(\overline{P}^+\) that represents \(\succeq_M\), we have
\[f \succeq_{E,M} g\]
iff for some \(h \in M\), \(f Eh \succeq_{MEh} g Eh\)
iff \(\text{reg}_{M,P^+|E}(f) \leq \text{reg}_{M,P^+|E}(g)\),
as desired.

6 Conclusion

We proposed an alternative belief representation using weighted sets of probabilities, and described a natural approach to updating in such a situation and a natural approach to determining the weights. We also showed how weighted sets of probabilities can be combined with regret to obtain a decision rule, MWER, and provided an axiomatization that characterizes static and dynamic preferences induced by MWER.

We have considered preferences indexed by menus here. Stoye [17] used a different framework: choice functions. A choice function maps every finite set \(M\) of acts to a subset \(M'\) of \(M\). Intuitively, the set \(M'\) consists of the ‘best’ acts in \(M\). Thus, a choice function gives less information than a preference order; it gives only the top elements of the preference order. The motivation for working with choice functions is that an agent can reveal his most preferred acts by choosing them when the menu is offered. In a menu-independent setting, the agent can reveal his whole preference order; to decide if \(f \succ g\), it suffices to present the agent with a choice among \(\{f, g\}\). However, with regret-based choices, the menu matters; the agent’s most preferred choice(s) when presented with \(\{f, g\}\) might no longer be the most preferred choice(s) when presented with a larger menu. Thus, a whole preference order is arguably not meaningful with regret-based choices. Stoye [17] provides a representation theorem for MER where the axioms are described in terms of choice functions. The axioms that we have attributed to Stoye are actually the menu-based analogue of his axioms. We believe that it should be possible to provide a characterization of MWER using choice functions, although we have not yet proved this.

Finally, we briefly considered the issue of dynamic consistency and consistent planning. As we showed, making this precise in the context of regret involves a number of subtleties. We hope to return to this issue in future work.

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References

