and Skolem functions were introduced, appropriately enough, by Skolem (1920). Oddly enough, it was Skolem who introduced the Herbrand universe (Skolem, 1928).

Herbrand's theorem (Herbrand, 1930) has played a vital role in the development of automated reasoning. Herbrand is also the inventor of unification. Gödel (1930) built on the ideas of Skolem and Herbrand to show that first-order logic has a complete proof procedure. Alan Turing (1936) and Alonzo Church (1936) simultaneously showed, using very different proofs, that validity in first-order logic was not decidable. The excellent text by Enderton (1972) explains all of these results in a rigorous yet understandable fashion.

Abraham Robinson proposed that an automated reasoner could be built using propositionalization and Herbrand's theorem, and Paul Gilmore (1960) wrote the first program. Davis and Putnam (1960) introduced the method of Section 9.1. Prawitz (1960) developed the key idea of letting the quest for propositional inconsistency drive the search, and generating terms from the Herbrand universe only when they were necessary to establish propositional inconsistency. After further development by other researchers, this idea led J. A. Robinson (no relation) to develop resolution (Robinson, 1965).

In AI, resolution was adopted for question-answering systems by Cordell Green and Bertram Raphael (1968). Early AI implementations put a good deal of effort into data structures that would allow efficient retrieval of facts; this work is covered in AI programming texts (Charniak et al., 1987; Norvig, 1992; Forbus and de Kleer, 1993). By the early 1970s, forward chaining was well established in AI as an easily understandable alternative to resolution. AI applications typically involved large numbers of rules, so it was important to develop efficient rule-matching technology, particularly for incremental updates. The technology for production systems was developed to support such applications. The production system language OPS-5 (Forgy, 1981; Brownston et al., 1985), incorporating the efficientrete match process (Forgy, 1982), was used for applications such as the R1 expert system for minicomputer configuration (McDermott, 1982).

The SOAR cognitive architecture (Laird et al., 1987; Laird, 2008) was designed to handle very large rule sets—up to a million rules (Doorenbos, 1994). Example applications of SOAR include controlling simulated fighter aircraft (Jones et al., 1998), airspace management (Taylor et al., 2007), AI characters for computer games (Wintermute et al., 2007), and training tools for soldiers (Wray and Jones, 2005).

The field of deductive databases began with a workshop in Toulouse in 1977 that brought together experts in logical inference and database systems (Gallaire and Minker, 1978). Influential work by Chandra and Harel (1980) and Ullman (1985) led to the adoption of Datalog as a standard language for deductive databases. The development of the magic sets technique for rule rewriting by Bancilhon et al. (1986) allowed forward chaining to borrow the advantage of goal-directedness from backward chaining. Current work includes the idea of integrating multiple databases into a consistent dataspace (Halevy, 2007).

Backward chaining for logical inference appeared first in Hewitt's PLANNER language (1969). Meanwhile, in 1972, Main Colmerauer had developed and implemented Prolog for the purpose of parsing natural language—Prolog's clauses were intended initially as context-free grammar rules (Roussel, 1975; Colmerauer et al., 1973). Much of the theoretical background for logic programming was developed by Robert Kowalski, working...
with Colmerauer; see Kowalski (1988) and Colmerauer and Roussel (1993) for a historical overview. Efficient Prolog compilers are generally based on the Warren Abstract Machine (WAM) model of computation developed by David H. D. Warren (1983). Van Roy (1990) showed that Prolog programs can be competitive with C programs in terms of speed.

Methods for avoiding unnecessary looping in recursive logic programs were developed independently by Smith et al. (1986) and Tamaki and Sato (1986). The latter paper also included memoization for logic programs, a method developed extensively as **tabled logic programming** by David S. Warren. Swift and Warren (1994) show how to extend the WAM to handle tabling, enabling Datalog programs to execute an order of magnitude faster than forward-chaining deductive database systems.

Early work on constraint logic programming was done by Jaffar and Lassez (1987). Jaffar et al. (1992) developed the CLP(R) system for handling real-valued constraints. There are now commercial products for solving large-scale configuration and optimization problems with constraint programming; one of the best known is ILOG (Junker, 2003). Answer set programming (Gelfond, 2008) extends Prolog, allowing disjunction and negation.

Texts on logic programming and Prolog, including Shoham (1994), Bratko (2001), Clocksin (2003), and Clocksin and Mellish (2003). Prior to 2000, the Journal of Logic Programming was the journal of record; it has now been replaced by Theory and Practice of Logic Programming. Logic programming conferences include the International Conference on Logic Programming (ICLP) and the International Logic Programming Symposium (ILPS).

Research into **mathematical theorem proving** began even before the first complete first-order systems were developed. Herbert Gelemter’s Geometry Theorem Prover (Gelernter, 1959) used heuristic search methods combined with diagrams for pruning false subgoals and was able to prove some quite intricate results in Euclidean geometry. The demodulation and paramodulation rules for equality reasoning were introduced by Wos et al. (1967) and Wos and Robinson (1968), respectively. These rules were also developed independently in the context of term-rewriting systems (Knuth and Bendix, 1970). The incorporation of equality reasoning into the unification algorithm is due to Gordon Plotkin (1972). Jouannaud and Kirchner (1991) survey equational unification from a term-rewriting perspective. An overview of unification is given by Baader and Snyder (2001).

A number of control strategies have been proposed for resolution, beginning with the unit preference strategy (Wos et al., 1964). The set-of-support strategy was proposed by Wos et al. (1965) to provide a degree of goal-directedness in resolution. Linear resolution first appeared in Loveland (1970). Genesereth and Nilsson (1987, Chapter 5) provide a short but thorough analysis of a wide variety of control strategies.

A Computational Logic (Boyer and Moore, 1979) is the basic reference on the Boyer-Moore theorem prover. Stickel (1992) covers the Prolog Technology Theorem Prover (PTTP), which combines the advantages of Prolog compilation with the completeness of model elimination. SETHEO (Letz et al., 1992) is another widely used theorem prover based on this approach. LEANTAP (Becket and Posega, 1995) is an efficient theorem prover implemented in only 25 lines of Prolog. Weidenbach (2001) describes SPASS, one of the strongest current theorem provers. The most successful theorem prover in recent annual competitions has been VAMPIRE (Riazanov and Voronkov, 2002). The COQ system (Bertot et al., 2004) and the E