total of $6^2 = 36$ elements.

We usually assume that dice are “fair,” namely that each of the six possibilities for a particular die has probability $\frac{1}{6}$, and that each of the 36 possible rolls in $\Omega$ has probability $\frac{1}{36}$. But we can also consider “loaded” dice in which there is a different distribution of probabilities. For example, let

$$
\begin{align*}
\Pr_1(\square) &= \Pr_1(\blacksquare) = \frac{1}{4}; \\
\Pr_1(\blacklozenge) &= \Pr_1(\lozenge) = \Pr_1(\clubsuit) = \frac{1}{8}.
\end{align*}
$$

Then $\sum_{d \in D} \Pr_1(d) = 1$, so $\Pr_1$ is a probability distribution on the set $D$, and we can assign probabilities to the elements of $\Omega = D^2$ by the rule

$$
\Pr_{1/1}(d,d') = \Pr_1(d) \cdot \Pr_1(d').
$$

(8.2)

For example, $\Pr_{1/1}(\square, m) = \frac{1}{4} \cdot \frac{1}{8} = \frac{1}{32}$. This is a valid distribution because

$$
\sum_{\omega \in \Omega} \Pr_{1/1}(\omega) = \sum_{d,d' \in D} \Pr_{1/1}(d,d') = \sum_{d \in D} \Pr_1(d) \sum_{d' \in D} \Pr_1(d') = 1 \cdot 1 = 1.
$$

We can also consider the case of one fair die and one loaded die,

$$
\Pr_{0/1}(d,d') = \Pr_0(d) \cdot \Pr_1(d'),
$$

(8.3)

in which case $\Pr_{0/1}(\square, m) = \frac{1}{6} \cdot \frac{1}{8} = \frac{1}{48}$. Dice in the “real world” can’t really be expected to turn up equally often on each side, because there is not perfect symmetry; but $\frac{1}{6}$ is usually pretty close to the truth.

An event is a subset of $\Omega$. In dice games, for example, the set

$$
\{ \square \cdot \blacksquare, \blacksquare \cdot \square, \blacklozenge \cdot \lozenge, \lozenge \cdot \blacklozenge, \clubsuit \cdot \diamondsuit, \diamondsuit \cdot \clubsuit \}
$$

is the event that “doubles are thrown.” The individual elements $\omega$ of $\Omega$ are called elementary events because they cannot be decomposed into smaller subsets; we can think of $\omega$ as a one-element event $\{ \omega \}$.

The probability of an event $A$ is defined by the formula

$$
\Pr(\omega \in A) = \sum_{\omega \in A} \Pr(\omega);
$$

(8.4)

and in general if $R(\omega)$ is any statement about $\omega$, we write ‘$\Pr(R)$’ for the sum of all $\Pr(\omega)$ such that $R(\omega)$ is true. Thus, for example, the probability of doubles with fair dice is

$$
\frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \frac{1}{18},
$$

but when both dice are loaded with probability distribution $\Pr_1$, it is

$$
\frac{1}{48} + \frac{1}{48} + \frac{1}{48} + \frac{1}{48} = \frac{3}{16} > \frac{1}{6}.
$$

Loading the dice makes the event “doubles are thrown” more probable.