total of $6^2 = 36$ elements.

We usually assume that dice are “fair,” namely that each of the six possibilities for a particular die has probability $\frac{1}{6}$, and that each of the 36 possible rolls in $\Omega$ has probability $\frac{1}{36}$. But we can also consider “loaded” dice in which there is a different distribution of probabilities. For example, let

$$
Pr_1(\square) = Pr_1(\square) = \frac{1}{4};
Pr_1(\bigstar) = Pr_1(\bigstar) = \frac{1}{8}.
$$

Then $\sum_{d\in D} Pr_1(d) = 1$, so $Pr_1$ is a probability distribution on the set $D$, and we can assign probabilities to the elements of $\Omega = D^2$ by the rule

$$
Pr_{11}(d d') = Pr_1(d) Pr_1(d').
$$

For example, $Pr_{11}(m \square) = \frac{1}{4} \cdot \frac{1}{8} = \frac{1}{32}$. This is a valid distribution because

$$
\sum_{\omega \in \Omega} Pr_{11}(\omega) = \sum_{d d' \in D^2} Pr_{11}(d d') = \sum_{d \in D} Pr_1(d) \sum_{d' \in D} Pr_1(d') = 1 \cdot 1 = 1.
$$

We can also consider the case of one fair die and one loaded die,

$$
Pr_{01}(d d') = Pr_0(d) Pr_1(d'),
$$
in which case $Pr_{01}(m \square) = \frac{1}{6} \cdot \frac{1}{8} = \frac{1}{48}$. Dice in the “real world” can’t really be expected to turn up equally often on each side, because there is not perfect symmetry; but $\frac{1}{6}$ is usually pretty close to the truth.

An event is a subset of $\Omega$. In dice games, for example, the set

$$
\{ \square \square, \square \bigstar, \bigstar \square, \bigstar \bigstar, \square \bigstar, \bigstar \square, \square \square \}
$$
is the event that “doubles are thrown.” The individual elements $\omega$ of $\Omega$ are called elementary events because they cannot be decomposed into smaller subsets; we can think of $\omega$ as a one-element event $\{\omega\}$.

The probability of an event $A$ is defined by the formula

$$
Pr(\omega \in A) = \sum_{\omega \in A} Pr(\omega);
$$

and in general if $R(\omega)$ is any statement about $\omega$, we write ‘$Pr(R(\omega))$’ for the sum of all $Pr(\omega)$ such that $R(\omega)$ is true. Thus, for example, the probability of doubles with fair dice is $\frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \frac{1}{6}$, but when both dice are loaded with probability distribution $Pr_1$ it is $\frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{3}{8} > \frac{1}{6}$. Loading the dice makes the event “doubles are thrown” more probable.